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1^a Prova: Cálculo Vetorial e Geometria Analítica

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1 (1,5 pts.) *Considere o ponto* $P = (0, -2, 1)$ *e o vetor* $\overrightarrow{v} = 2\overrightarrow{i} + \overrightarrow{j} - 3\overrightarrow{k}$. *Determine um ponto* Q *de modo que* $\overrightarrow{PQ} = -2\overrightarrow{v}$.

2 (2,0 pts.) *Considere um triângulo de vértices* P, Q *e* R, ΔPQR, *seja* S *um ponto no lado* PQ *tal que* $\overrightarrow{PS} = 3\overrightarrow{SQ}$. *Escreva o vetor* $\overrightarrow{RS}$ *como combinação linear dos vetores* $\overrightarrow{RP}$ *e* $\overrightarrow{QR}$.

3 (2,5 pts.) *Verifique que os vetores* $\overrightarrow{u} = 2\overrightarrow{i} + \overrightarrow{j} - 2\overrightarrow{k}$, $\overrightarrow{v} = \overrightarrow{i} + \overrightarrow{j} - \overrightarrow{k}$ *e* $\overrightarrow{w} = -\overrightarrow{i} + \overrightarrow{j} - 2\overrightarrow{k}$ *formam uma base para* $\mathbb{R}^3$. *Determine as coordenadas do vetor* $\overrightarrow{a} = \overrightarrow{i} - 3\overrightarrow{j} + 5\overrightarrow{k}$ *nesta base.*

4 (2,5 pts.) *Decomponha o vetor* $\overrightarrow{w} = [-1, -3, 2]$ *como soma de dois vetores* $\overrightarrow{u}$ *e* $\overrightarrow{v}$ *com* $\overrightarrow{u}$ *paralelo ao vetor* $[0, 1, 3]$ *e* $\overrightarrow{v}$ *ortogonal a* $[0, 1, 3]$.

5 (2,5 pts.) *Determine as coordenadas do vetor* $\overrightarrow{v} = [x, y, z]$ *sabendo que* $\overrightarrow{v} \cdot (2\overrightarrow{i} + 3\overrightarrow{j}) = 6$ *e* $\overrightarrow{v} \times (2\overrightarrow{i} + 3\overrightarrow{j}) = 4\overrightarrow{k}$.

CVGA : Prova 1

① $P = (0,-2,1)$, $\vec{v} = 2\vec{i} + \vec{j} - 3\vec{k}$.

Determinar Q de modo que $\vec{PQ} = -2\vec{v}$

Solução:

Suponha $Q = (x,y,z)$.

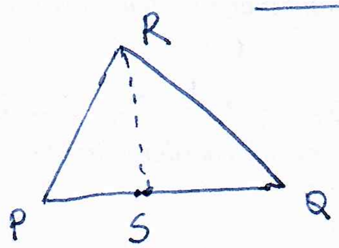
$$\vec{PQ} = [x, y+2, z-1]$$

$$\vec{PQ} = -2\vec{v} \Rightarrow [x, y+2, z-1] = -2[2,1,-3]$$

$$\Rightarrow \begin{cases} x = -4 \\ y+2 = -2 \\ z-1 = -3 \end{cases}$$

$$\therefore \quad x = -4, \quad y = -4, \quad z = -2$$

Daí $\quad Q = (-4,-4,-2)$

② 

$$\vec{PS} = 3\vec{SQ}$$

$$\vec{RS} = \alpha\,\vec{RP} + \beta\,\vec{QR}$$

$\alpha = ?$, $\beta = ?$

$$\vec{RS} = \vec{RP} + \vec{PS} = \vec{RP} + 3\vec{SQ} \qquad \vec{SQ} = \vec{SR} + \vec{RQ}$$

$$= \vec{RP} + 3(\vec{SR} + \vec{RQ}) = \vec{RP} + 3\vec{SR} + 3\vec{RQ}$$

$$= \vec{RP} - 3\vec{RS} - 3\vec{QR}$$

$$\Rightarrow \quad 4\vec{RS} = \vec{RP} - 3\vec{QR}$$

Daí, $\quad \vec{RS} = \frac{1}{4}\vec{RP} - \frac{3}{4}\vec{QR}$

③ $\vec{u} = 2\vec{i} + \vec{j} - 2\vec{k}$, $\vec{v} = \vec{i} + \vec{j} - \vec{k}$, $\vec{w} = -\vec{i} + \vec{j} - 2\vec{k}$

$\{\vec{u}, \vec{v}, \vec{w}\}$ é base de $\mathbb{R}^3$?

$\vec{a} = \vec{i} - 3\vec{j} + 5\vec{k}$. coordenadas de $\vec{a}$ na base $\{\vec{u}, \vec{v}, \vec{w}\}$

Solução

Basta verificar que $\vec{u}$, $\vec{v}$ e $\vec{w}$ são l.i. (são 3 vetores)

$$\det\begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & 1 \\ -2 & -1 & -2 \end{pmatrix} = -4+1-2-(2-2-2) = -3 \neq 0$$

$\therefore$ $\vec{u}, \vec{v}$ e $\vec{w}$ são l.i logo formam uma base p/ $\mathbb{R}^3$.

$$\vec{a} = x\vec{u} + y\vec{v} + z\vec{w} . \quad (x=?, y=?, z=?)$$

$$i - 3j + 5k = x(2\vec{i} + \vec{j} - 2\vec{k}) + y(\vec{i} + \vec{j} - \vec{k}) + z(-\vec{i} + \vec{j} - 2\vec{k})$$

$$= (2x + y - z)\vec{i} + (x + y + z)\vec{j} + (-2x - y - 2z)\vec{k}$$

$$\Rightarrow \begin{cases} 2x + y - z = 1 \\ x + y + z = -3 \\ -2x - y - 2z = 5 \end{cases}$$

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & 1 & 1 & -3 \\ -2 & -1 & -2 & 5 \end{array}\right) \sim \left(\begin{array}{ccc|c} 2 & 1 & -1 & 1 \\ 1 & 1 & 1 & -3 \\ 0 & 0 & -3 & 6 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & +1 & -3 \\ 2 & 1 & -1 & 1 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

$(z = -2)$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & -3 \\ 0 & -1 & -3 & -5 \\ 0 & 0 & 1 & -2 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & -2 & -8 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & -12 \\ 0 & 1 & 0 & 11 \\ 0 & 0 & 1 & -2 \end{array}\right)$$

$\therefore\ z=-2,\ y=11,\ x=-12$

Daí $\underbrace{\vec{a} = -12\vec{u} + 11\vec{v} - 2\vec{w}}$

(4) $\vec{w} = [-1,-3,2]$. Escrever $\vec{w} = \vec{u} + \vec{v}$

com $\vec{u} \parallel [0,1,3]$ e $\vec{v} \perp [0,1,3]$

Solução

$\vec{u} \parallel [0,1,3] \Rightarrow \vec{u} = t[0,1,3]$

Supondo $\vec{v} = [x,y,z]$, $\vec{v} \perp [0,1,3] \Rightarrow \underline{y+3z=0}$

$\vec{w} = \vec{u} + \vec{v} \Rightarrow [-1,-3,2] = t[0,1,3] + [x,y,z]$

$= [x, t+y, 3t+z]$

$$\Rightarrow \begin{cases} x=-1 \\ t+y=-3 \\ 3t+z=2 \end{cases}$$

$\underline{x=-1}$

$y+3z=0 \Rightarrow \underline{y=-3z}$

$\begin{cases} t=-3-y=-3+3z \\ 3t+z=2 \Rightarrow 3t=2-z \end{cases}$

$\Rightarrow -9+9z = 2-z$

$\Rightarrow 10z = 11$

$\boxed{z = 11/10}$

$y=-3z=-3\cdot\frac{11}{10} \Rightarrow \boxed{y=\frac{-33}{10}}$

$t=-3-y=-3+\frac{33}{10}=3/10$

$\boxed{t=3/10}$

Daí,

$$\vec{u} = [0, 3/10, 9/10]$$

$$\vec{v} = \left[-1, \frac{-33}{10}, \frac{11}{10}\right]$$

$$[1,-3,2] = \left[0, \frac{3}{10}, \frac{9}{10}\right] + \left[-1, -\frac{33}{10}, \frac{11}{10}\right]$$

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⑤ $\vec{v} = [x, y, z]$

$$\left\{\begin{array}{l} \vec{v} \cdot (2\vec{i} + 3\vec{j}) = 6 \\ \vec{v} \times (2\vec{i} + 3\vec{j}) = 4\vec{k} \end{array}\right.$$

Determinar as coord. de $\vec{v}$.

Solução

$$\vec{v} \cdot (2\vec{i} + 3\vec{j}) = 6 \Rightarrow 2x + 3y = 6$$

$$\vec{v} \times (2\vec{i} + 3\vec{j}) = \det\begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ x & y & z \\ 2 & 3 & 0 \end{pmatrix} = -3z\vec{i} + 2z\vec{j} + (3x - 2y)\vec{k}$$

$$\vec{v} \times (2\vec{i} + 3\vec{j}) = 4\vec{k} \Rightarrow -3z\vec{i} + 2z\vec{j} + (3x - 2y)\vec{k} = 4\vec{k}$$

$$\Rightarrow \left\{\begin{array}{l} z = 0 \\ 3x - 2y = 4 \end{array}\right. \qquad z = 0$$

$$\left.\begin{array}{l} 3x + 2y = 4 \\ 2x + 3y = 6 \end{array}\right| \Rightarrow x = 0,\ y = 2$$

$$\therefore \quad \vec{v} = [0, 2, 0]$$