EXISTENCE OF GROUND STATES FOR A QUASILINEAR COUPLED SYSTEM IN $\mathbb{R}^N$

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Abstract. In this work we consider the following class of quasilinear coupled systems

$$(S_\theta)\begin{cases} -\Delta u + a(x)u - \Delta(u^2)u = g(u) + \theta\lambda(x)|u|^{\alpha-2}u|v|^{\beta}, & x \in \mathbb{R}^N,\\ -\Delta v + b(x)v - \Delta(v^2)v = h(v) + \theta\lambda(x)|v|^{\beta-2}v|u|^{\alpha}, & x \in \mathbb{R}^N,\end{cases}$$

where $N \geq 3$ and $a, b : \mathbb{R}^N \to \mathbb{R}$ are positive potentials, $\lambda : \mathbb{R}^N \to \mathbb{R}$ is a suitable continuous function, $\theta > 0$ and $\alpha, \beta > 2$ satisfying $\alpha + \beta < 2.2^*$. On the nonlinear terms we assume that g, h are in C^1 class and are superlinear functions at infinity and at the origin. The main theorem is stated without the well known Ambrosetti-Rabinowitz condition at infinity. Using a change of variable, we turn the quasilinear coupled system into a nonlinear coupled system, where we establish a variational approach based on Nehari method.

1. Introduction

We look for ground states for the general class of quasilinear coupled systems involving Schrödinger equations (S_θ). This class of systems imposes some difficulties. The first one is that the energy functional associated to System (S_θ) is not well defined in the whole space $H^1(\mathbb{R}^N)^2$. Thus, motivated by seminal works [1–6] we also use a change of variable to reformulate our initial problem, obtaining a nonlinear coupled system. After change of variable, the modified problem has an associated energy functional well defined in the whole space $H^1(\mathbb{R}^N)^2$ and the solutions are related with solutions of the initial System (S_θ). The second difficulty is the lack of compactness due to the fact that the system is defined in the whole Euclidean space $\mathbb{R}^N$. Moreover, System (S_θ) involve strongly coupled Schrödinger equations because of the coupling terms in the right hand side. We emphasize that we do not use the well known Ambrosetti-Rabinowitz condition. We suppose that the potentials a, b satisfy the following hypotheses:

(a_0) $a, b, \lambda \in C(\mathbb{R}^N, \mathbb{R})$ are 1-periodic functions for each $x_1, x_2, ..., x_N$;

(a_1) $a(x) \geq a_0$ and $b(x) \geq b_0$ for some $a_0, b_0 > 0$

(a_2) $\lambda(x) \geq 0$ for all $x \in \mathbb{R}^N$ and $\lambda(x) > 0$ for all $x \in \Omega$, for some $\Omega \subset \mathbb{R}^N$ such that $|\Omega| < +\infty$.

(g_0) $g, h \in C^1(\mathbb{R}, \mathbb{R})$;

(g_1) $|g(t)| \leq C\left(1 + |t|^{p-1}\right), \quad |h(t)| \leq C\left(1 + |t|^{p-1}\right), \quad$ for all $t \in \mathbb{R}$ for some $C > 0$ and $p \in (4, 2.2^*)$

(g_2) $\lim_{t\to 0} \dfrac{g(t)}{t} = 0, \quad \lim_{t\to 0} \dfrac{h(t)}{t} = 0;$

(g_3) $\lim_{|t|\to +\infty} \dfrac{g(t)}{t^3} = +\infty, \quad \lim_{|t|\to +\infty} \dfrac{h(t)}{t^3} = +\infty;$

(g_4) The functions $t \to \dfrac{g(t)}{t^3}, \quad t \to \dfrac{h(t)}{t^3}$ are strictly increasing in $|t|$.

(g_5) There holds $0 \leq G(t) \leq G(|t|)$ and $0 \leq H(t) \leq H(|t|)$, for all $t \in \mathbb{R}$.

Theorem 1.1. *Suppose that (a_0)-(a_2) and (g_0)-(g_5) hold. Then, there exists $\theta_0 > 0$ such that System (S_θ) has at least one positive ground state solution, for all $\theta \geq \theta_0$.*

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