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On a Schrödinger equation in a generalized electrodynamics

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We consider a system deriving from the interaction of the Schrödinger equation with a modified version of the Maxwell equation proposed by B. Podolsky in 1942. The advantage of the Maxwell lagrangian proposed by Podolsky, in contrast e.g. to the Born-Infeld Lagrangian, is that the equation involving the electromagnetic field is still linear.

More specifically the lagrangian is given by (see [1, Formula (3.9)]):

$$\begin{aligned}\mathcal{L}_{\mathrm{BP}} &= \frac{1}{8\pi}\left\{|\mathbf{E}|^2 - |\mathbf{H}|^2 + a^2\left[(\mathrm{div}\mathbf{E})^2 - \left(\nabla\times\mathbf{H} - \frac{1}{c}\dot{\mathbf{E}}\right)^2\right]\right\}\\ &= \frac{1}{8\pi}\left\{|\nabla\phi + \frac{1}{c}\partial_t\mathbf{A}|^2 - |\nabla\times\mathbf{A}|^2 + a^2\left[\left(\Delta\phi + \frac{1}{c}\mathrm{div}\partial_t\mathbf{A}\right)^2 - \left(\nabla\times\nabla\times\mathbf{A} + \frac{1}{c}\partial_t(\nabla\phi + \frac{1}{c}\partial_t\mathbf{A})\right)^2\right]\right\}\end{aligned}$$

where

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\partial_t\mathbf{A} \qquad \mathbf{H} = \nabla\times\mathbf{A}$$

is the electromagnetic field. Here $a > 0$ is a constant. Of course, whenever $a = 0$ it reduces to the Lagrangian of the classical electrodynamics.

After using the "minimal coupling rule" which describes the interaction between the Schrödinger matter field and the electromagnetic field driven by $\mathcal{L}_{\mathrm{BP}}$, the search of standing waves solutions in the purely electrostatic situation led to a system of this type in $\mathbb{R}^3$

$$\begin{cases} -\Delta u + u + q\phi u = |u|^{p-2}u \\ -\Delta\phi + \Delta^2\phi = qu^2. \end{cases}$$

where $q > 0$ has the meaning of the electric charge and all the other physical constants have been normalized. The parameter p is subcritical.

Observe that, for every fixed $u \in H^1(\mathbb{R}^3)$, the second equation has a unique solution ϕ_u in a suitable Hilbert space, however it is not homogeneous like in the classical Schrödinger-Poisson system. In particular it seems difficult to work with "Nehari methods". In spite of this, we are able, by using variational methods (in particular Mountain Pass arguments), to prove the existence of solutions, depending on p and on the value of the charge $q > 0$.

Joint work with Collaborator1 (Institution1) and Collaborator2 (Institution2).

References

[1] PODOLSKY, B. - A Generalized Electrodynamics. *Physical Review*, **62**, 68-71, 1942.

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