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Existence of multiple solutions for the van der Waals-Allen-Cahn equation

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I will discuss the existence of multiple solutions for the following nonlinear problem: for a fixed $V \in \mathbb{R}^+ =]0, +\infty[$ and $\varepsilon > 0$ small, find $u \in H_0^1(\Omega)$, and $\lambda \in \mathbb{R}$ such that

$$-\varepsilon^2 \Delta u + W'(u) = \lambda,$$

and

$$\int_\Omega u(x)\,\mathrm{d}x = V,$$

where Ω is an open bounded set in $\mathbb{R}^N$ and $W : \mathbb{R} \to \mathbb{R}$ is a function of class C^2 which satisfies the following assumptions:

(a) $W(0) = W'(0) = 0$, $W''(0) > 0$;

(b) there exists $s_0 \in]0, +\infty[$ such that

$$W(s_0) = \min \{W(s) : s \in \mathbb{R}\} < 0;$$

(c) suitable growth conditions.

The simplest example of this type of potentials is given by the non-symmetric Allen-Cahn potential:

$$W(s) = s^2(s - s_1)(s - s_2),$$

where $0 < s_1 < s_0 < s_2$. In theoretical biology equations of this type model pattern formation related to solutions which are not absolute minima of the energy. From a purely mathematical viewpoint, the above equation is also interesting due to its relation with the theory of constant mean curvature hypersurfaces.eory.

Joint work with Vieri Benci (Pisa University) and Stefano Nardulli (Federal University of Rio de Janeiro).

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