Existence and non-existence of positive solutions of quasi-linear elliptic equations involving gradient terms.

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Abstract

We study the existence and non-existence of non negative solutions in the whole Euclidean space of coercive quasi-linear and fully nonlinear elliptic equations described by

$$\Delta_p u = f(u) \pm g(|\nabla u|)$$

where

$$f \in C([0,\infty)), g \in C^{0,1}([0,\infty)) \text{ are strictly increasing with } f(0) = g(0) = 0.$$

We give conditions on f and g which guarantee the existence or absence of positive solutions of this problem in $\mathbb{R}^n$. These results represent a generalization to a result obtained for the case of the Laplacian operator, by Patricio Felmer, Alexander Quaas and Boyan Sirakov.

In the particular case of the problem with plus sign on the right hand side we obtain generalized Keller- Osserman integral conditions. It turns out that different conditions are needed when $p \geq 2$ or $p \leq 2$ to deal with the existence results. The existence and non-existence in this case are established in a weak sense (the Sobolev sense).

For the problem with minus sign we show the existence also independently of the operator whenever possible to ensure the non-negativity of the non-linearity. The result of non-existence in this case is obtained independently of the gradient term.

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