MINIMAL HYPERSURFACES AND THE ALLEN-CAHN EQUATION ON CLOSED MANIFOLDS

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Abstract: Since the late 70s parallels between the theory of phase transitions and critical points of the area functional have helped us to understand variational properties of solutions to semilinear elliptic PDEs of the form

$$-\varepsilon\Delta u + W'(u)/\varepsilon = 0 \tag{1}$$

for $u : M \to \mathbb{R}$ defined in a Riemannian manifold M, where W is a double-well potential, and spaces of hypersurfaces which minimize the area in an appropriate sense. We will discuss some recent developments in this direction which extend well-known analogies regarding minimizers of the associated energy functional to more general variational solutions.

Borrowing ideas from the min-max theory of minimal hypersurfaces, we construct variational solutions of equation (1) – known as the *Allen-Cahn equation* – in a closed manifold, and study the asymptotic growth of the corresponding critical values as well as solutions with least non-trivial energy. This is joint work with M.A.M. Guaraco.

Furthermore we obtain an upper bound for the stability index of the minimal hypersurfaces which arise from solutions with uniformly bounded energy and index as $\varepsilon \downarrow 0$ in terms of the Morse index of these solutions by comparing the second inner variation of the energy functional to the second variation of the area.