On Lane-Emden systems with singular nonlinearities and applications to MEMS

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Abstract

We analyse the Lane-Emden system

$$\begin{cases} -\Delta u = \dfrac{\lambda f(x)}{(1-v)^2} & \text{in} \quad \Omega \\ -\Delta v = \dfrac{\mu g(x)}{(1-u)^2} & \text{in} \quad \Omega \\ 0 \leq u, v < 1 & \text{in} \quad \Omega \\ u = v = 0 & \text{on} \quad \partial\Omega \end{cases} \tag{$S_{\lambda,\mu}$}$$

where λ and μ are positive parameters and Ω is a smooth bounded domain of $\mathbb{R}^N$ $(N \geq 1)$. Here we prove the existence of a critical curve Γ which splits the positive quadrant of the (λ, μ)-plane into two disjoint sets $\mathcal{O}_1$ and $\mathcal{O}_2$ such that the problem $(S_{\lambda,\mu})$ has a smooth minimal stable solution (u_λ, v_μ) in $\mathcal{O}_1$, while for $(\lambda, \mu) \in \mathcal{O}_2$ there are no solutions of any kind. We also establish upper and lower estimates for the critical curve Γ and regularity results on this curve if $N \leq 7$. Our proof is based on a delicate combination involving maximum principle and L^p estimates for semi-stable solutions of $(S_{\lambda,\mu})$.

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