COEXISTENCE STATES IN A CROSS-DIFFUSION SYSTEM OF A PREDATOR-PREY MODEL

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Abstract

In this work we study the existence and non-existence of coexistence states of the following Lotka-Volterra predator-prey system with cross-diffusion

$$\begin{cases} -\nabla\left[\dfrac{1}{R(v)}\nabla u - \dfrac{uR'(v)\nabla v}{R(v)[R(v)+g(v)]}\right] = u(\lambda - u + bv) & \text{in } \Omega, \\ -d_v\Delta v = v(\mu - v - cu) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega. \end{cases} \tag{0.1}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a bounded domain with a smooth boundary, $d_v, b, c > 0$ are positive constants, $R, g : [0,\infty) \to \mathbb{R}$ are functions of class $\mathcal{C}^{2,\gamma}$ and $\mathcal{C}^{1,\gamma}$, $0 < \gamma < 1$, respectively, such that

$$R(0) > 0 \quad \text{and} \quad g(s),\ R'(s) > 0,\ C \geq R(s)\ \forall s \in [0,\infty),$$

for some positive constant C. The above system was proposed in [1] and it is a particular version of the original model proposed in [2]. From the ecological point of view the functions u and v denote the population densities of the predator and prey, respectively, the left sides of the equations for u and v are dispersal (or diffusion) terms: the first term contains the dispersal term of predator and the second one incorporates the dispersal term of prey, they describe the spatial movement of species. The right sides are the classical predator-prey Lotka-Volterra reaction functions. The term $uR'(v)/(R(v)[R(v)+g(v)]$ is called cross-diffusion term, where R describes the turning rate of predator and g describes the predator satiation. This is the main contribution of [2] in order to obtain a more realistic model to describe the behaviour of predator-prey.

We use mainly bifurcation methods and a priori bounds. Actually, we extend the bifurcation result of [3] (see also [4]) for semilinear systems to the quasilinear case (0.1). Thus, we give conditions on the parameters λ and μ to ensure existence and non-existence of coexistence states. We also compare our results with the classical linear diffusion predator-prey model. Our results suggest that when there is no abundance of prey, the predator needs to be a good hunter to survive.

Referências

[1] C. Cosner, Reaction-diffusion-advection models for the effects and evolution of dispersal, *Discrete Contin. Dyn. Syst.* **34** (2014) 1701–1745.

[2] P. Kareiva and G. Odell, Swarms of predators exhibit 'preytaxis' if individual predators use area-restricted search, *The American Naturalist* **130** (1987) 233–270.

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[3] J. López-Gómez, Nonlinear eigenvalues and global bifurcation application to the search of positive solutions for general Lotka-Volterra reaction diffusion systems with two species, *Differential Integral Equations* **7** (1994) 1427–1452.

[4] J. López-Gómez, Spectral Theory and Nonlinear Function Analysis, (Chapman & Hall/CRC 2001).