Geometry and qualitative theory for PDEs in non-Euclidean environments

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Abstract

The study of Partial Differential Equations in infinite dimension spaces or in Riemannian manifolds is a central theme of research within the modern mathematical analysis. Indeed, a number of physical and economic models have been successfully described by PDEs in such non-Euclidean environments. From the mathematical point of view, this modern theory has fostered the development of revolutionary analytic and geometric tools, powerful enough to have deep applications to a number of correlated fields of mathematics. We will start off our talk giving a general overview about nonlinear elliptic equations modeled in Hilbert spaces. Particular attention will be given to a recent work we developed with A. Swiech (Georgia Tech) about existence and regularity theory for elliptic PDEs with obstacles in Hilbert spaces. Subsequently, we will turn out attention to free boundary problems in Riemannian manifolds. We will restrict ourselves to the program, in development with Lei Zhang (Univ. Florida), of developing fine regularity tools naturally requested in the study of free boundary problems, such as monotonicity formulas, for equations modeled in general Riemannian manifolds. At the end, we shall talk a bit about asymptotic behavior of solutions to Yamabe-type fully-nonlinear PDEs near singular points. This work, recently carried out in collaboration with Zheng-Chao Han (Rutgers Univ.) and YanYan Li (Rutgers Univ., is of particular interest to the study of geometric PDEs. In particular to existence results of complete metrics in punctuated spheres with positive and constant σ_k curvature that are conformally equivalent to the usual metric.