
7. Técnicas de Integração



7.1 Mudança de Variável (método de substituição)

01. $\int e^{5x} dx$ (resp. $e^{5x}/5 + C$)
02. $\int \operatorname{sen} ax dx$ (resp. $-\frac{1}{a} \cos ax + C, a \neq 0$)
03. $\int \frac{dx}{\operatorname{sen}^2(3x)}$ (resp. $-\frac{1}{3} \cotg 3x + C$)
04. $\int \frac{dx}{3x-7}$ (resp. $\frac{1}{3} |\ln 3x-7| + C$)
05. $\int \tan 2x dx$ (resp. $-\frac{1}{2} \ln |\cos 2x| + C$)
06. $\int \cot(5x-7) dx$ (resp. $\frac{1}{5} \ln |\operatorname{sen}(5x-7)| + C$)
07. $\int \tan x \sec^2 x dx$ (resp. $\frac{1}{2} \operatorname{tg}^2 x + C$)
08. $\int (\cot e^x) e^x dx$ (resp. $\ln |\operatorname{sen} e^x| + C$)
09. $\int \operatorname{sen}^2 x \cos x dx$ (resp. $\frac{1}{3} \operatorname{sen}^3 x + C$)
10. $\int \cos^3 x \operatorname{sen} x dx$ (resp. $-\frac{1}{4} \cos^4 x + C$)
11. $\int x\sqrt{x^2+1} dx$ (resp. $\frac{1}{3} (x^2+1)^{3/2} + C$)
12. $\int \frac{x^2 dx}{\sqrt{x^3+1}}$ (resp. $\frac{2}{3} \sqrt{x^3+1} + C$)
13. $\int \frac{\cos x dx}{\operatorname{sen}^2 x}$ (resp. $-(\operatorname{sen} x)^{-1} + C$)
14. $\int \frac{dx}{\cos^2 x \sqrt{\tan x-1}}$ (resp. $2\sqrt{\operatorname{tg} x-1} + C$)
15. $\int \frac{\ln(x+1) dx}{x+1}$ (resp. $\frac{1}{2} [\ln(x+1)]^2 + C$)
16. $\int \frac{\cos x dx}{\sqrt{2 \operatorname{sen} x+1}}$ (resp. $\sqrt{2 \operatorname{sen} x+1} + C$)

17. $\int \frac{\sqrt{\tan x + 1}}{\cos^2 x} dx$ (resp. $\frac{2}{3} (\operatorname{tg} x + 1)^{3/2} + C$)
18. $\int \frac{\cos 2x dx}{(2 + 3 \operatorname{sen} 2x)^3}$ (resp. $-\frac{1}{12 (2 + 3 \operatorname{sen} 2x)^2} + C$)
19. $\int \frac{\arcsin x dx}{\sqrt{1 - x^2}}$ (resp. $\frac{1}{2} \operatorname{arcsen}^2 x + C$)
20. $\int \frac{\arctan x dx}{1 + x^2}$ (resp. $\frac{1}{2} \operatorname{arctg}^2 x + C$)
21. $\int \frac{\arccos^2 x dx}{\sqrt{1 - x^2}}$ (resp. $-\frac{1}{3} \arccos^3 x + C$)
22. $\int \frac{(x + 1) dx}{x^2 + 2x + 3}$ (resp. $\frac{1}{2} \ln (x^2 + 2x + 3) + C$)
23. $\int \frac{\cos x dx}{2 \operatorname{sen} x + 3}$ (resp. $\frac{1}{2} \ln (2 \operatorname{sen} x + 3) + C$)
24. $\int \frac{dx}{x \ln x}$ (resp. $\ln |\ln x| + C$)
25. $\int \frac{dx}{(1 + x^2) \arctan x}$ (resp. $\ln |\operatorname{arctg} x| + C$)
26. $\int \frac{\tan^3 x}{\cos^2 x} dx$ (resp. $\frac{1}{4} \operatorname{tg}^4 x + C$)
27. $\int \frac{dx}{\sqrt{1 - x^2} \arcsin x}$ (resp. $\ln |\operatorname{arcsen} x| + C$)
28. $\int \frac{\cos (\ln x)}{x} dx$ (resp. $\operatorname{sen} (\ln |x|) + C$)
29. $\int x a^{x^2} dx, a > 0$ (resp. $a^{x^2} / 2 \ln a + C$)
30. $\int 3^x e^x dx$ (resp. $e^x 3^x / (1 + \ln 3) + C$)
31. $\int e^{-3x} dx$ (resp. $-\frac{1}{3} e^{-3x} + C$)
32. $\int (e^{5x} + a^{5x}) dx, a > 0$ (resp. $\frac{1}{5} e^{5x} + \frac{1}{5 \ln a} a^{5x} + C$)
33. $\int e^{x^2 + 4x + 3} (x + 2) dx$ (resp. $\frac{1}{2} e^{x^2 + 4x + 3} + C$)
34. $\int \frac{(a^x - b^x)^2}{a^x b^x} dx$ (resp. $-2x + [(a/b)^x - (b/a)^x] / \ln (a/b) + C$)
35. $\int \frac{\exp (2x) dx}{2 + \exp (2x)}$ (resp. $\frac{1}{2} \ln (2 + e^{2x})$)
36. $\int \frac{dx}{\sqrt{1 - 3x^2}}$ (resp. $\frac{\sqrt{3}}{3} \operatorname{arcsen}(\sqrt{3}x) + C$)

37. $\int \frac{dx}{\sqrt{16-9x^2}}$ (resp. $\frac{1}{3} \arcsen(3x/4) + C$)
38. $\int \frac{dx}{9x^2+4}$ (resp. $\frac{1}{6} \arctg(3x/2) + C$)
39. $\int \frac{x dx}{\sqrt{1-x^4}}$ (resp. $\frac{1}{2} \arcsen(x^2) + C$)
40. $\int \frac{x dx}{x^4+a^4}$ (resp. $\frac{1}{2a^2} \arctg(x^2/a^2) + C$)
41. $\int \frac{e^x dx}{\sqrt{1-e^{2x}}}$ (resp. $\arcsen e^x + C$)
42. $\int \frac{\cos x dx}{a^2 + \sen^2 x}$ (resp. $\frac{1}{a} \arctg[(\sen x)/a] + C$)
43. $\int \frac{dx}{x\sqrt{1-\ln^2 x}}$ (resp. $\arcsen(\ln x) + C$)
44. $\int \frac{(x - \arctan x) dx}{1+x^2}$ (resp. $\frac{1}{2} \ln(1+x^2) - \frac{1}{2} \arctg^2 x + C$)
45. $\int \frac{\sqrt{1+\sqrt{x}} dx}{\sqrt{x}}$ (resp. $\frac{4}{3} (1+\sqrt{x})^{3/2} + C$)
46. $\int \frac{dx}{\sqrt{x}\sqrt{1+\sqrt{x}}}$ (resp. $4(1+\sqrt{x})^{1/2} + C$)
47. $\int \frac{e^x dx}{1+e^{2x}}$ (resp. $\arctg(e^x) + C$)
48. $\int \frac{\cos x dx}{\sqrt[3]{\sen^2 x}}$ (resp. $3\sqrt{\sen x} + C$)
49. $\int \frac{\sen 2x dx}{\sqrt{1+\cos^2 x}}$ (resp. $-2(1+\cos^2 x)^{1/2} + C$)
50. $\int \frac{\cos^3 x dx}{\sen^4 x}$ (resp. $(\sen x)^{-1} - \frac{1}{3}(\sen x)^{-3} + C$)
51. $\int \frac{\sqrt[3]{\tan^2 x} dx}{\cos^2 x}$ (resp. $\frac{3}{5}(\tg x)^{5/3} + C$)
52. $\int \frac{dx}{2\sen^2 x + 3\cos^2 x}$ (resp. $\frac{1}{\sqrt{6}} \arctg(\sqrt{\frac{2}{3}} \tg x) + C$)

7.2 Integração por Partes

01. $\int x e^x dx$ (resp. $x e^x - e^x + C$)
02. $\int x \ln x dx$ (resp. $\frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$)
03. $\int x \operatorname{sen} x dx$ (resp. $\operatorname{sen} x - x \cos x + C$)
04. $\int \ln x dx$ (resp. $x \ln x - x + C$)
05. $\int \arcsen x dx$ (resp. $x \arcsen x + \sqrt{1-x^2} + C$)
06. $\int x^n \ln x dx$ (resp. $\frac{x^{n+1}}{n+1} (\ln x - \frac{1}{n+1}) + C$)
07. $\int x \arctan x dx$ (resp. $\frac{1}{2} x^2 \operatorname{arctg} x - \frac{1}{2} x + \frac{1}{2} \operatorname{arctg} x + C$)
08. $\int \ln(x^2 + 1) dx$ (resp. $x \ln(1+x^2) - 2x + 2 \operatorname{arctg} x + C$)
09. $\int \frac{\arcsen \sqrt{x}}{\sqrt{x}} dx$ (resp. $2\sqrt{x} \arcsen \sqrt{x} + 2\sqrt{1-x} + C$)
10. $\int \arcsin \left(\sqrt{\frac{x}{x+1}} \right) dx$ (resp. $x \arcsen \sqrt{\frac{x}{x+1}} - \sqrt{x} + \operatorname{arctg} \sqrt{x} + C$)
11. $\int \frac{x \arctan x dx}{(x^2 + 1)^2}$ (resp. $\frac{x - (1-x^2) \operatorname{arctg} x}{4(1+x^2)} + C$)
12. $\int x \arctan \left(\sqrt{x^2 - 1} \right) dx$ (resp. $\frac{1}{2} \left[x^2 \operatorname{arctg} \sqrt{x^2 - 1} - \sqrt{x^2 - 1} \right] + C$)
13. $\int \frac{\arcsen x dx}{x^2}$ (resp. $\ln \left| \frac{1 - \sqrt{1-x^2}}{x} \right| - \frac{\arcsen x}{x} + C$)
14. $\int \ln \left(x + \sqrt{1+x^2} \right) dx$ (resp. $x \ln \left(x + \sqrt{1+x^2} \right) - \sqrt{1+x^2} + C$)
15. $\int x^2 e^x dx$ (resp. $x^2 e^x - 2x e^x + 2e^x$)

7.3 Problemas de Valor Inicial

01. $\frac{ds}{dt} = 12t(3t^2 - 1)^3, s(1) = 3$ (resp. $s(t) = \frac{1}{2}(3t^2 - 1)^4 - 5$)
02. $\frac{d^2 y}{dx^2} = \sec^2 x, y(0) = 1, y'(0) = 0$ (resp. $y(t) = -\ln(|\cos t|) + 1$)
03. $\frac{dy}{dt} = e^t \operatorname{sen}(e^t - 2), y(\ln 2) = 0$ (resp. $y(t) = -\cos(e^t - 2) + 1$)

7.4 Decomposição em Frações Parciais

01. $\int \frac{(2x-1)}{(x-1)(x-2)} dx$ (resp. $-\ln|x-1| + 3\ln|x-2| + C$)
02. $\int \frac{xdx}{(x+1)(x+3)(x+5)}$ (resp. $-\frac{1}{8}\ln|x+1| + \frac{3}{4}\ln|x+3| - \frac{5}{8}\ln|x+5| + C$)
03. $\int \frac{(x^5+x^4-8)dx}{x^3-4x}$ (resp. $4x + 2\ln|x| + \frac{1}{2}x^2 + \frac{1}{3}x^3 + 5\ln|x-2| - 3\ln|x+2| + C$)
04. $\int \frac{x^4 dx}{(x^2-1)(x+2)}$ (resp. $\frac{1}{2}x^2 - 2x + \frac{1}{2}\ln\frac{|x-1|^{1/3}}{|x+1|} + \frac{16}{3}\ln|x+2| + C$)
05. $\int \frac{dx}{(x-1)^2(x-2)}$ (resp. $\frac{1}{x-1} + \ln\left|\frac{x-2}{x-1}\right| + C$)
06. $\int \frac{(x-8)dx}{x^3-4x^2+4x}$ (resp. $-2\ln|x| + \frac{3}{x-2} + 2\ln|x-2| + C$)
07. $\int \frac{(3x+2)dx}{x(x+1)^3}$ (resp. $2\ln|x| - \frac{1}{2(x+1)^2} + \frac{2}{x+1} - 2\ln|x+1| + C$)
08. $\int \frac{dx}{x^3+1}$ (resp. $\frac{1}{3}\ln\frac{x+1}{(x^2-x+1)^{1/2}} + \frac{1}{\sqrt{3}}\arctan(\frac{2x-1}{\sqrt{3}}) + C$)
09. $\int \frac{x^5}{x^3-1}$ (resp. $\frac{1}{3}x^3 + \frac{1}{3}\ln|x^3-1| + C$)
10. $\int \frac{(x^2+2x+1)dx}{(1+x^2)^2}$ (resp. $-\frac{1}{1+x^2} + \arctan|x| + C$)
11. $\int \frac{dx}{x^2+2x+5}$ (resp. $\frac{1}{2}\arctan(\frac{x+1}{2}) + C$)
12. $\int \frac{dx}{3x^2-2x+4}$ (resp. $\frac{1}{\sqrt{11}}\arctan(\frac{3x-1}{\sqrt{11}}) + C$)
13. $\int \frac{dx}{x^2-6x+5}$ (resp. $\frac{1}{4}\ln\left|\frac{x-5}{x-1}\right| + C$)
14. $\int \frac{dx}{2x^2-2x+1}$ (resp. $\arctan(2x-1) + C$)
15. $\int \frac{(6x-7)dx}{3x^2-7x+11}$ (resp. $\ln|3x^2-7x+11| + C$)
16. $\int \frac{(7x+1)dx}{6x^2+x-1}$ (resp. $\frac{1}{2}\ln|2x+1| + \frac{2}{3}\ln|3x-1|$)
17. $\int \frac{(3x-2)dx}{5x^2-3x+2}$ (resp. $\frac{3}{10}\ln|5x^2-3x+2| - \frac{11}{5\sqrt{31}}\arctan(\frac{10x-3}{\sqrt{31}})$)
18. $\int \frac{(6x^4-5x^3+4x^2)dx}{2x^2-x+1}$ (resp. $x^3 - \frac{x^2}{2} + \frac{1}{4}\ln|2x^2-x+1| + \frac{1}{2\sqrt{7}}\arctan(\frac{4x-1}{\sqrt{7}})$)

7.5 Integrais Trigonômétricas

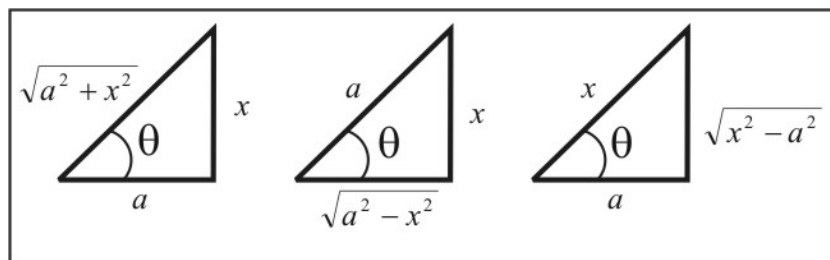
01. $\int \sin^3 x dx$ (resp. $-\cos x + \frac{1}{3} \cos^3 x + C$)
02. $\int \sin^5 x dx$ (resp. $-\cos x + \frac{2}{3} \cos^3 x - \frac{1}{5} \cos^5 x + C$)
03. $\int \cos^4 x \sin^3 x dx$ (resp. $-\frac{1}{5} \cos^5 x + \frac{1}{7} \cos^7 x + C$)
04. $\int \frac{\cos^3 x dx}{\sin^4 x}$ (resp. $\csc x - \frac{1}{3} \csc^3 x + C$)
05. $\int \tan^3 x dx$ (resp. $\frac{1}{2} \tan^2 x + \log |\cos x| + C$)
06. $\int \cot^5 x dx$ (resp. $-\frac{1}{4} \csc^4 x + \csc^2 x + \log |\sin x| + C$)
07. $\int \cos^4 x dx$ (resp. $\frac{1}{4} \sin x \cos^3 x + \frac{3}{8} \cos x \sin x + \frac{3}{8} x + C$)
08. $\int \tan^2 x dx$ (resp. $\tan x - x + C$)
09. $\int \frac{\cos^{20} x}{\sin^{22} x} dx$ (resp. $-\frac{1}{21} \cot^{21} x + C$)
10. $\int \cos 4x \cos 7x dx$ (resp. $\frac{1}{6} \sin 3x + \frac{1}{22} \sin 11x + C$)
11. $\int \cos^2 x \sin^3 x dx$ (resp. $\frac{1}{5} \cos^5 x - \frac{1}{3} \cos^3 x + C$)
12. $\int \sqrt[3]{\cos x} \sin^5 x dx$ (resp. $-\frac{3}{16} \cos^{\frac{16}{3}} x + \frac{3}{5} \cos^{\frac{10}{3}} x - \frac{3}{4} \cos^{\frac{4}{3}} x + C$)
13. $\int \sqrt{\sin x} \cos x dx$ (resp. $\frac{2}{3} \sin^{\frac{3}{2}} x + C$)
14. $\int \frac{\cot^3 x}{\sqrt{\sin x}} dx$ (resp. $-\frac{2}{5(\sqrt{\sin x})^5} + \frac{2}{\sqrt{\sin x}} + C$)
15. $\int \sin^p x \cos^3 x dx$ (resp. $\frac{1}{1+p} (\sin x)^{1+p} - \frac{1}{3+p} (\sin x)^{3+p} + C$)
16. $\int \sin^2 x dx$ (resp. $-\frac{1}{2} \cos x \sin x + \frac{1}{2} x + C$)
17. $\int \sec x dx$ (resp. $\ln |\sec x + \tan x|$).
18. $\int \sec^3 x dx$ (resp. $\frac{\sin x \sec^2 x}{2} + \frac{\ln |\sec x + \tan x|}{2} + C$).

(em (17) faça $\sec x = \frac{\sec x + \tan x}{\sec x + \tan x} \sec x$ e $u = \sec x + \tan x$ e em (18) escreva $\sec^3 x dx = \sec x \sec^2 x dx$)

19. $\int \cos^2 x dx$ (resp. $\frac{1}{2} \cos x \sin x + \frac{1}{2}x$)
20. $\int \sin^2 x \sec^3 x dx$ (resp. $\frac{1}{2} \sin^3 x \sec^2 x + \frac{1}{2} \sin x - \frac{1}{2} \ln |\sec x + \tan x| + C$)
21. $\int \sin^2 x \cos^2 x dx$ (resp. $-\frac{1}{4} \sin x \cos^3 x + \frac{1}{8} \cos x \sin x + \frac{1}{8}x + C$)
22. $\int \cos^6 x dx$ (resp. $\cos x \sin x (\frac{1}{6} \cos^4 x + \frac{5}{24} \cos^2 x + \frac{5}{16}) + \frac{5}{16}x + C$)
23. $\int \operatorname{cosec}^4 x dx$ (resp. $-\cot x - \frac{1}{3} \cot^3 x + C$. Faça $\csc^4 x = \csc^2 x + \csc^2 \cot^2 x$)
24. $\int \tan^4 x \sec^4 x dx$ (resp. $\frac{\tan^7 x}{7} + \frac{\tan^5 x}{5} + C$)
25. $\int \cos 2x \sin 4x dx$ (resp. $-\frac{1}{12} \cos 6x - \frac{1}{4} \cos 2x + C$)
26. $\int \sin^4 x dx$ (resp. $-\frac{1}{2} \cos x \sin x + \frac{1}{2}x + C$)
27. $\int \sec^8 x dx$ (resp. $\frac{\tan^7 x}{7} + \frac{3 \tan^5 x}{5} + \tan^3 x + \tan x + C$)
28. $\int \sin x \sin 3x dx$ (resp. $\frac{1}{4} \sin 2x - \frac{1}{8} \sin 4x + C$)
29. $\int \frac{\cos^2 x}{\sin^4 x} dx$ (resp. $-\frac{1}{3} \cot^3 x + C$)
30. $\int \sin(\frac{x}{4}) \cos(\frac{3x}{4}) dx$ (resp. $-\frac{1}{2} \cos x + \cos \frac{1}{2}x + C$)

Substituições Trigonômétricas

As integrais que envolvem as expressões $\sqrt{x^2 \pm a^2}$ e $\sqrt{a^2 \pm x^2}$ podem ser calculadas com auxílio de substituições especiais. As substituições mais comuns são: $x = a \operatorname{tg} \theta$, $x = a \operatorname{sen} \theta$ e $x = a \operatorname{sec} \theta$ e elas são deduzidas a partir das relações nos triângulos retângulos da figura abaixo:



Substituições Trigonômétricas

- | | | |
|---|-----------------------|--------------------------------------|
| 1. a mudança $x = a \operatorname{tg} \theta$ | substitui $a^2 + x^2$ | por $a^2 \sec^2 \theta$ |
| 2. a mudança $x = a \operatorname{sen} \theta$ | substitui $a^2 - x^2$ | por $a^2 \cos^2 \theta$ |
| 3. a mudança $x = a \sec \theta$ | substitui $x^2 - a^2$ | por $a^2 \operatorname{tg}^2 \theta$ |

01. $\int \frac{\sqrt{a^2 - x^2} dx}{x^2}$ (resp. $-\frac{\sqrt{a^2 - x^2}}{x} - \arcsin(x/a) + C$)
02. $\int \frac{x^2}{\sqrt{4 - x^2}} dx$ (resp. $-\frac{1}{2}x\sqrt{4 - x^2} + 2 \arcsin \frac{1}{2}x + C$)
03. $\int \frac{dx}{\sqrt{(a^2 + x^2)^3}}$ (resp. $\frac{x}{a^2\sqrt{a^2 + x^2}} + C$)
04. $\int \frac{xdx}{\sqrt{4x^2 + 8x + 5}}$ (resp. $\frac{1}{4}\sqrt{4x^2 + 8x + 5} - \frac{1}{2} \ln(2x + \sqrt{4x^2 + 8x + 5} + 2) + C$)
05. $\int \frac{dx}{\sqrt{x^2 - 4x - 5}}$ (resp. $\ln(x - 2 + \sqrt{x^2 - 4x - 5}) + C$)
06. $\int \frac{dx}{\sqrt{x^2 + 2x + 2}}$ (resp. $\ln|x + 1 + \sqrt{1 + (x + 1)^2}| + C$)
07. $\int \frac{xdx}{\sqrt{6x - x^2 - 5}}$ (resp. $-\sqrt{6x - x^2 - 5} + 3 \arcsin(\frac{1}{2}x - \frac{3}{2}) + C$)
08. $\int \frac{dx}{\sqrt{1 + x^2}}$ (resp. $\ln|x + \sqrt{1 + x^2}| + C$)
09. $\int \frac{dx}{\sqrt{x^2 - 4}}$ (resp. $\ln(x + \sqrt{x^2 - 4}) + C$)
10. $\int \frac{dx}{\sqrt{4x^2 - 4x - 3}}$ (resp. $\frac{1}{2} \ln(2x - 1 + \sqrt{4x^2 - 4x - 3}) + C$)
11. $\int x^2\sqrt{4 - x^2} dx$ (resp. $2 \arcsin(x/2) - \frac{1}{8}x(4 - x^2)^{3/2} - \frac{1}{8}x^3\sqrt{4 - x^2} + C$)
12. $\int \frac{x}{\sqrt{8 - x^2 - 2x}} dx$ (resp. $-\sqrt{8 - x^2 - 2x} - \arcsin(\frac{x+1}{3}) + C$)
13. $\int \frac{dx}{x^2\sqrt{x^2 + 9}}$ (resp. $-\frac{1}{9x}\sqrt{x^2 + 9} + C$)
14. $\int \frac{dx}{x^2\sqrt{x^2 - 16}}$ (resp. $\frac{1}{16x}\sqrt{x^2 - 16} + C$)
15. $\int \frac{\sqrt{x^2 - a^2} dx}{x}$ (resp. $\sqrt{-a^2 + x^2} - a \operatorname{arcsec}(x/a) + C$)
16. $\int \frac{dx}{\sqrt{2 - 3x - 4x^2}}$ (resp. $\frac{1}{2} \arcsin[(8x + 3)/\sqrt{41}]$)

17. $\int \frac{dx}{\sqrt{1+x+x^2}}$ (resp. $\ln|x+\frac{1}{2}+\sqrt{1+x+x^2}|$)
18. $\int \frac{dx}{\sqrt{5-7x-3x^2}}$ (resp. $\frac{1}{\sqrt{3}} \arcsin[(6x+7)/\sqrt{109}]$)
19. $\int \frac{dx}{\sqrt{x(3x+5)}}$ (resp. $\frac{1}{\sqrt{3}} \ln|6x+5+\sqrt{12x(3x+5)}|$)
20. $\int \frac{dx}{\sqrt{5x^2-x-1}}$ (resp. $\frac{1}{\sqrt{5}} \ln|10x-1+\sqrt{20(5x^2-x-1)}|$)
21. $\int \frac{(x+3)dx}{\sqrt{4x^2+4x+3}}$ (resp. $\frac{1}{4}\sqrt{4x^2+4x+3} + \frac{5}{4} \ln|2x+1+\sqrt{4x^2+4x+3}|$)
22. $\int \frac{(x-3)dx}{\sqrt{3+66x-11x^2}}$ (resp. $-\frac{1}{11}\sqrt{3+66x-11x^2}$)
23. $\int \frac{(3x+5)dx}{\sqrt{x(2x-1)}}$ (resp. $\frac{3}{2}\sqrt{x(2x-1)} + \frac{23}{4\sqrt{2}} \ln|4x-1+\sqrt{8x(2x-1)}|$)

7.6 Integral de Funções Especiais

Funções Envolvendo exp(x)

01. $\int \tanh x dx$ 02. $\int \frac{e^x}{e^x-1} dx$ 03. $\int \frac{e^x}{e^{2x}+1} dx$ 04. $\int \frac{1+\sinh x}{1+\cosh x} dx$
 [resp.: 01) $\ln(\cosh x)$ 02) $\ln(e^x-1)$ 03) $\arctan(e^x)$ 04) $\tanh \frac{x}{2} - \ln(\tanh \frac{x}{2} - 1) - \ln(\tanh \frac{x}{2} + 1)$]

Funções Envolvendo Radicais

01. $\int \frac{\sqrt{x} dx}{\sqrt[4]{x^3}+1}$ 02. $\int \frac{(\sqrt{x^3}-\sqrt[3]{x}) dx}{6\sqrt[4]{x}}$ 03. $\int \frac{(\sqrt[6]{x}+1) dx}{\sqrt[6]{x^7}+\sqrt[4]{x^5}}$
 04. $\int \frac{(\sqrt[7]{x}+\sqrt{x}) dx}{\sqrt[7]{x^8}+\sqrt[14]{x^{15}}}$ 05. $\int \frac{\sqrt{1+\sqrt[3]{x}} dx}{\sqrt[3]{x^2}}$ 06. $\int \frac{\sqrt{2-\sqrt[3]{x}} dx}{\sqrt[3]{x}}$
 [resp.: 01) $\frac{4}{3}[\sqrt[4]{x^3}-\ln(\sqrt[4]{x^3}+1)]$ 02) $\frac{2}{27}\sqrt[4]{x^9}-\frac{2}{13}\sqrt[12]{x^{13}}$ 03) $-\frac{6}{\sqrt[6]{x}}+\frac{12}{\sqrt[12]{x}}+2\log x-24\log(\sqrt[12]{x}+1)$
 04) $14\left[\sqrt[14]{x}-\frac{1}{2}\sqrt[7]{x}+\frac{1}{3}\sqrt[14]{x^3}-\frac{1}{4}\sqrt[7]{x^2}+\frac{1}{5}\sqrt[14]{x^5}\right]$ 05) $2(1+\sqrt{x})^{\frac{3}{2}}$ 06) $\frac{2}{5}(4+3\sqrt[3]{x})(2-\sqrt[3]{x})^{\frac{3}{2}}$]