UNIVERSIDADE FEDERAL DA PARAÍBA
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Cálculo I - Prova 2 - 07/11/2022

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Nome: ______________________________ Matrícula: ________________

1 (6,0 pts.) *Calcule a derivada ($f'(x)$) das funções abaixo:*

$$(a)\quad f(x) = \left(x^2+5x\right)^{x^3}, \qquad (b)\quad f(x) = \frac{1}{x}\cos^2(4x+3)$$

$$(c)\quad f(x) = 3^x \tan(x^2), \qquad (d)\quad f(x) = \sqrt[5]{\frac{x^2-x}{2x-1}}$$

$$(e)\quad f(x) = \left(5x^3+2x\right)^{23} \qquad (f)\quad f(x) = \frac{\cos(3x+1)}{x^2+3}.$$

2 (2,0 pts.) *Determine a equação da reta tangente à curva dada por $y = 3x^2 - 2x$ que é perpendicular à reta $y = -x$*

3 (2,0 pts.) *Suponha que a equação $F(x,y) = 0$ abaixo define, em cada caso, y como função de x. Calcule $\frac{dy}{dx}$.*

$$(a)\quad F(x,y) = \sqrt{2x} - \sqrt{3y}, \qquad (b)\quad F(x,y) = xy - \cos(xy).$$

Boa Prova !!

Cálculo I Prova 2

~~2021.2~~

2022.1

① (a) $f(x) = (x^2+5x)^{x^3}$

$$\ln f(x) = x^3 \ln(x^2+5x)$$

$$\frac{f'(x)}{f(x)} = 3x^2 \ln(x^2+5x) + x^3 \cdot \left(\frac{2x+5}{x^2+5x}\right)$$

$$\Rightarrow f'(x) = (x^2+5x)^{x^3}\left[3x^2 \ln(x^2+5x) + x^3\,\frac{2x+5}{x^2+5x}\right]$$

(b) $f(x) = \frac{1}{x}\cos^2(4x+3)$

$$f'(x) = \frac{-1}{x^2}\cos^2(4x+3) + \frac{1}{x}\cdot 2\cos(4x+3)\cdot(-\operatorname{sen}(4x+3))$$

$$f'(x) = -\frac{\cos^2(4x+3)}{x^2} - \frac{8}{x}\cos(4x+3)\operatorname{sen}(4x+3)$$

(c) $f(x) = 3^x \cdot \operatorname{tg}(x^2)$

$$y = 3^x$$
$$\ln y = x \cdot \ln 3$$
$$y'/y = \ln 3$$
$$y' = 3^x \cdot \ln 3$$

$$f'(x) = 3^x \ln 3 \cdot \operatorname{tg} x^2 + 3^x \cdot \sec^2(x^2)\cdot 2x$$

$$f'(x) = 3^x \ln 3 \cdot \operatorname{tg}(x^2) + 2x\, 3^x \sec^2(x^2)$$

(d) $f(x) = \sqrt[5]{\frac{x^2-x}{2x-1}}$

$$f'(x) = \frac{1}{5}\left(\frac{x^2-x}{2x-1}\right)^{-4/5}\cdot\left[\frac{(2x-1)^2 - 2(x^2-x)}{(2x-1)^2}\right]$$

(e) $f(x) = (5x^3+2x)^{23}$

$$f'(x) = 23(5x^3+2x)^{22}\cdot(15x^2+2)$$

(f) $f(x) = \dfrac{\cos(3x+1)}{x^3+3}$

$$f'(x) = \frac{-\operatorname{sen}(3x+1)\cdot 3(x^3+3) - 3x^2\cos(3x+1)}{(x^3+3)^2}$$

$$\boxed{f'(x) = \frac{-3(x^3+3)\operatorname{sen}(3x+1) - 3x^2\cos(3x+1)}{(x^3+3)^2}}$$

2. Reta tg a $y = 3x^2-2x$ perpend ~~paralela~~ à reta $y=-x$

$y' = 6x-2$ $\quad 6x-2 = 1 \Rightarrow \boxed{x = 1/2}$

$x = 1/2 \Rightarrow y = 3\cdot\frac{1}{4} - 2\cdot 1/2 = \frac{3}{4} - 1 = -1/4$

$\Rightarrow y = -1/4$

Reta Tg $\quad y + 1/4 = 1\,(x - 1/2)$

$$\boxed{y = x - 3/4}$$

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(a) $F(x,y) = \sqrt{2x} - \sqrt{3y}$

$$F(x,y)=0 \Rightarrow \frac{1}{\cancel{2}\sqrt{2x}}\cancel{2} - \frac{1}{2\sqrt{3y}}\cdot 3y' = 0$$

$$\Rightarrow \frac{1}{\sqrt{2x}} - \frac{3y'}{2\sqrt{3y}} = 0$$

$$\Rightarrow y' = \frac{-1/\sqrt{2x}}{3}\left(-2\sqrt{3y}\right)$$

$$\Rightarrow \boxed{y' = \frac{+2\sqrt{3y}}{3\sqrt{2x}}}$$

(b) $F(x,y) = xy - \cos(xy)$

$$xy - \cos(xy) = 0 \Rightarrow y + xy' + \operatorname{sen}(xy)\left[y + xy'\right] = 0$$

$$\Rightarrow y + y\operatorname{sen}(xy) + xy' + xy'\operatorname{sen}(xy) = 0$$

$$\Rightarrow y'(x + x\operatorname{sen}(xy)) + y + y\operatorname{sen}(xy) = 0$$

$$\Rightarrow y' = \frac{-y(1+\operatorname{sen}(xy))}{x(1+\operatorname{sen}(xy))}$$

$$\boxed{y' = -y/x}$$