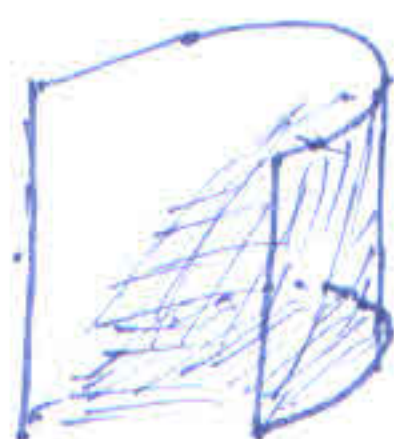
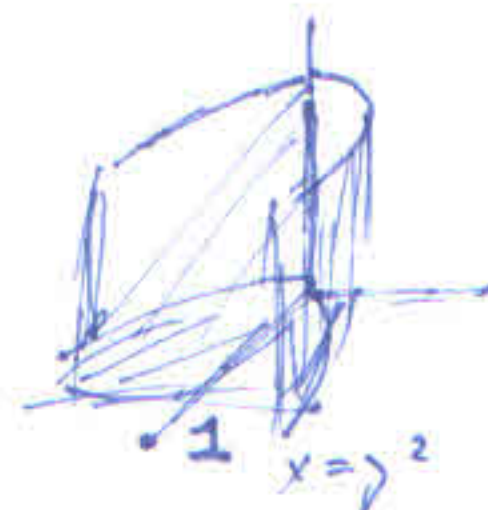
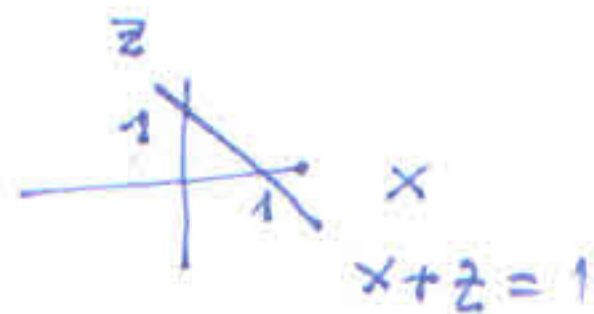
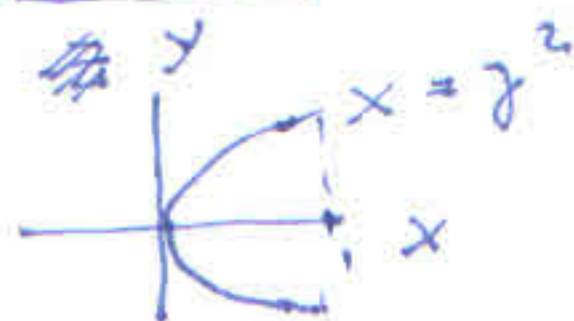


① Determine o volume do sólido $W \subset \mathbb{R}^3$, onde

(a) W é limitado pelo cilindro $x = y^2$ e os planos $z = 0$ e $x + z = 1$

Solução:



$$W: \begin{cases} 0 \leq x \leq 1 \\ -\sqrt{x} \leq y \leq \sqrt{x} \\ 0 \leq z \leq 1-x \end{cases}$$

$$\text{vol}(W) = \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} \int_0^{1-x} dz dy dx$$

$$= \int_0^1 \int_{-\sqrt{x}}^{\sqrt{x}} (1-x) dy dx$$

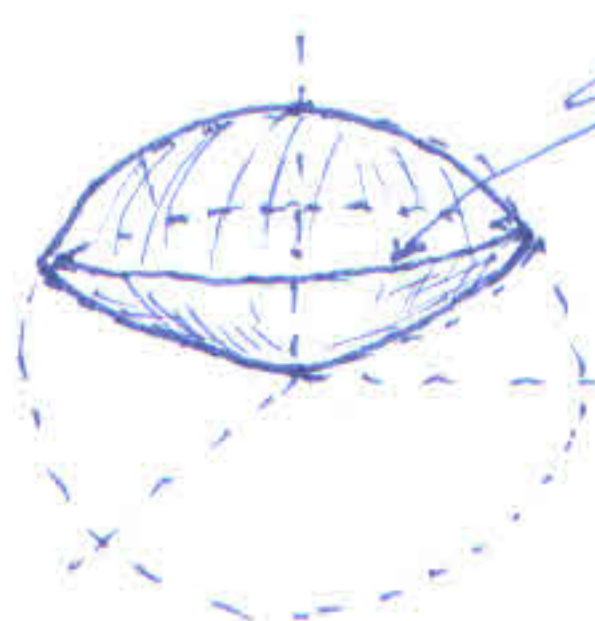
$$= \int_0^1 (1-x) 2\sqrt{x} dx$$

$$\text{vol}(W) = \int_0^1 (2\sqrt{x} - 2x^{3/2}) dx = \left(\frac{4}{3} x^{3/2} - \frac{4}{5} x^{5/2} \right) \Big|_0^1$$

$$= \frac{4}{3} - \frac{4}{5} = \frac{8}{15} \text{ unidades de volume.}$$

(c) W é limitado pela esfera $x^2 + y^2 + z^2 = 4$ e pelo parabolóide $x^2 + y^2 = 3z$

Solução:



$$\begin{cases} z = 1 \\ x^2 + y^2 = 3 \end{cases}$$

Em coordenadas cilíndricas W é descrito como:

$$W: \begin{cases} 0 \leq r \leq \sqrt{3} \\ 0 \leq \theta \leq 2\pi \\ \frac{r^2}{3} \leq z \leq \sqrt{4-r^2} \end{cases}$$

$$W: \begin{cases} 0 \leq r \leq \sqrt{3} \\ 0 \leq \theta \leq 2\pi \\ \frac{r^2}{3} \leq z \leq \sqrt{4-r^2} \end{cases}$$

$$\text{vol}(W) = \int_0^{\sqrt{3}} \int_0^{2\pi} \int_{r^2/3}^{\sqrt{4-r^2}} r \, dz \, d\theta \, dr = \int_0^{\sqrt{3}} \int_0^{2\pi} \left(r\sqrt{4-r^2} - \frac{r^3}{3} \right) d\theta \, dr$$

$$= 2\pi \int_0^{\sqrt{3}} \left(r\sqrt{4-r^2} - \frac{r^3}{3} \right) dr = 2\pi \left(\frac{1}{2} \sqrt{4-r^2} - \frac{r^3}{3} \right) \Big|_0^{\sqrt{3}}$$

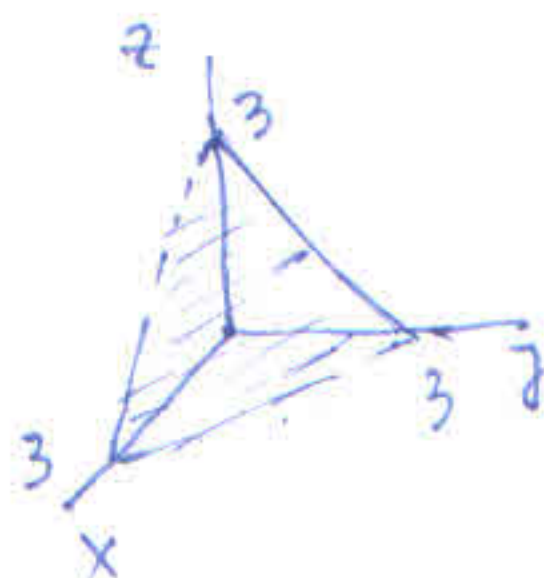
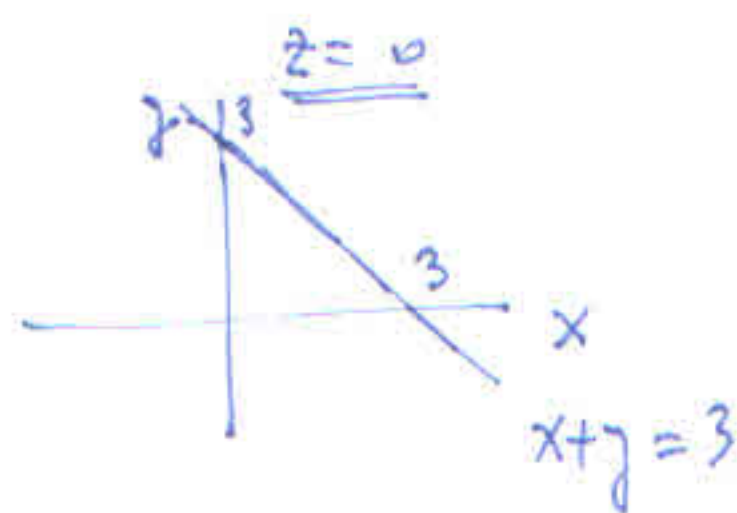
$$= 2\pi \left(-\frac{1}{3} (4-r^2)^{3/2} - \frac{r^4}{12} \right) \Big|_0^{\sqrt{3}} = 2\pi \left(-\frac{16}{12} + \frac{4}{3} \right)$$

$$= 2\pi \left(-\frac{4}{3} + \frac{8}{3} \right) = \frac{8\pi}{3} \text{ u.v.}$$

② Calcule $\iiint_W f \, dv$, onde:

(a) $f(x, y, z) = x - y$ e W é o tetraedro limitado pelos planos coord. e pelo plano $x + y + z = 3$

Solução



$$W: \begin{cases} 0 \leq x \leq 3 \\ 0 \leq y \leq 3-x \\ 0 \leq z \leq 3-(x+y) \end{cases}$$

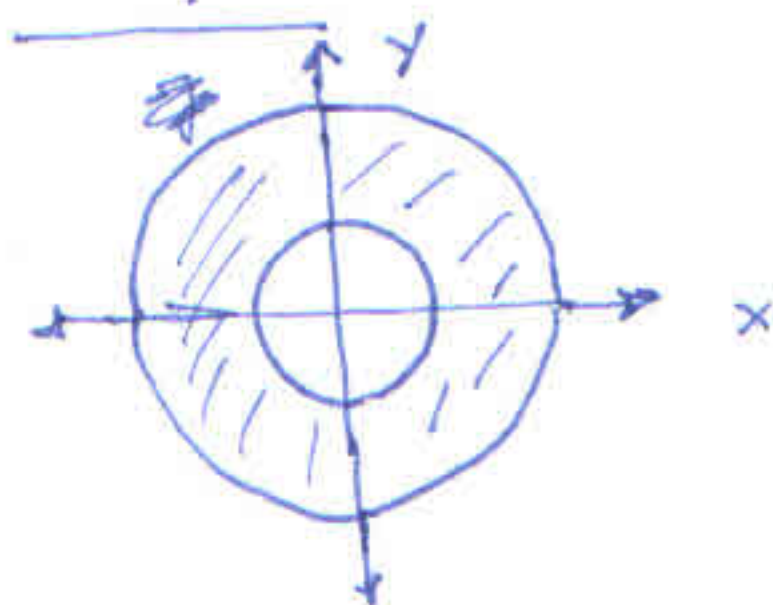
$$\iiint_W f \, dv = \int_0^3 \int_0^{3-x} \int_0^{3-(x+y)} (x-y) \, dz \, dy \, dx$$

$$= \int_0^3 \int_0^{3-x} [3(x-y) - (x+y)(x-y)] \, dy \, dx = \int_0^3 \int_0^{3-x} (-x^2 + y^2 + 3x - 3y) \, dy \, dx$$

$$\begin{aligned}
 \iiint_W f dv &= \int_0^3 \left(-x^2 y + \frac{y^3}{3} + 3xy - \frac{3y^2}{2} \right) \Big|_{y=0}^{y=3-x} dx \\
 &= \int_0^3 \left[-x^2(3-x) + \frac{(3-x)^3}{3} + 3x(3-x) - \frac{3(3-x)^2}{2} \right] dx \\
 &= \int_0^3 \left\{ x(3-x)^2 + \frac{1}{3}(3-x)^3 - \frac{3}{2}(3-x)^2 \right\} dx \\
 &= \int_0^3 \left(\frac{2}{3}x - \frac{1}{2} \right) (3-x)^2 dx \\
 &= \dots = \underline{\underline{0}}
 \end{aligned}$$

(d) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$, W é a coroa esférica limitada por $x^2 + y^2 + z^2 = 1$ e $x^2 + y^2 + z^2 = 4$

Solução



Em coord esféricas W é descrito por:

$$W: \begin{cases} 1 \leq \rho \leq 2 \\ 0 \leq \theta \leq 2\pi \\ 0 \leq \phi \leq \pi \end{cases} \quad \begin{aligned} dx dy dz &= \rho^2 \sin \phi \\ x^2 + y^2 + z^2 &= \rho^2 \end{aligned}$$

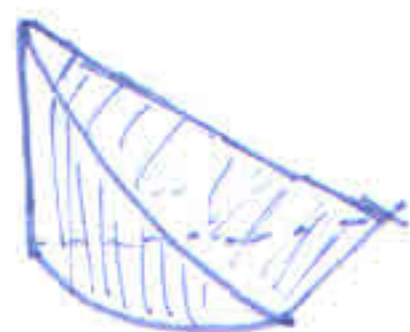
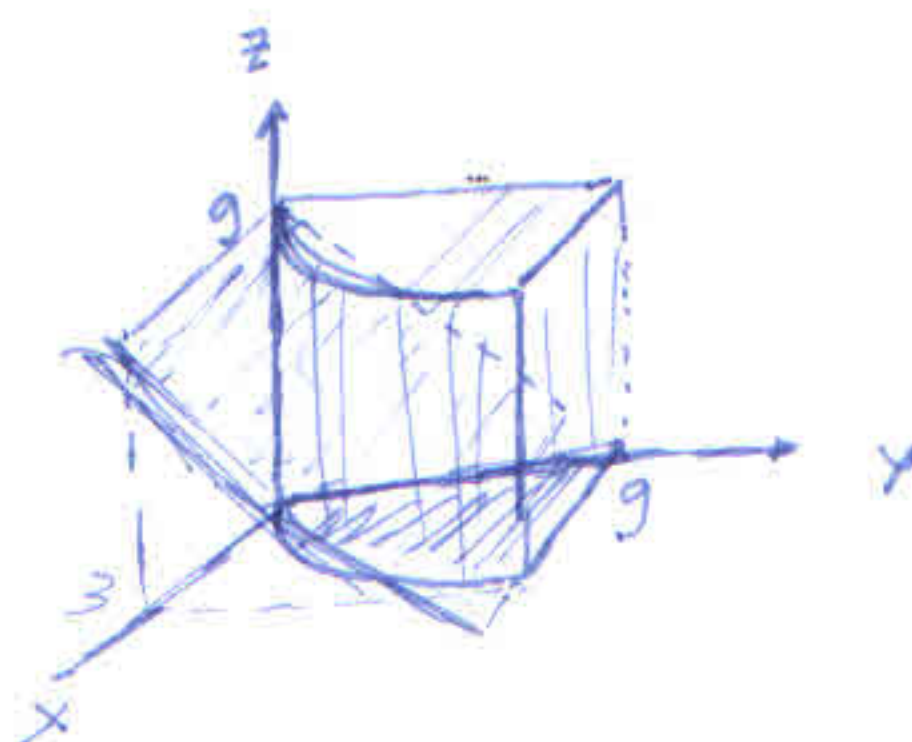
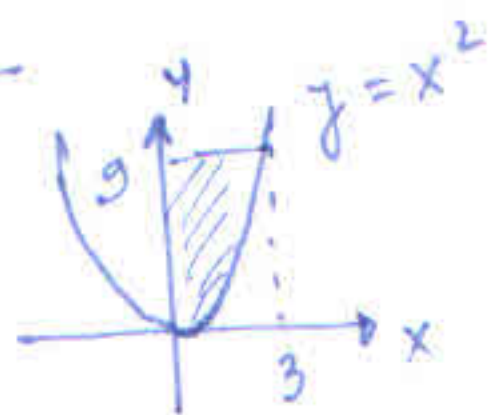
$$\begin{aligned}
 \iiint_W f dv &= \int_1^2 \int_0^{2\pi} \int_0^\pi \rho \cdot \rho^2 \sin \phi \, d\phi \, d\theta \, d\rho \\
 &= \int_1^2 \int_0^{2\pi} -\rho^3 \cos \phi \Big|_0^\pi d\theta \, d\rho = \int_1^2 \int_0^{2\pi} 2\rho^3 \, d\theta \, d\rho
 \end{aligned}$$

$$\int_1^2 \int_0^{2\pi} 2 \rho^3 d\theta d\rho = 4\pi \int_1^2 \rho^3 d\rho = \pi \rho^4 \Big|_1^2$$

$$= 16\pi - \pi = \underline{\underline{15\pi}}$$

- ③ Determine a massa de W . W é o sólido, no primeiro octante, limitado por $y = x^2$, $y = 9$, $z = 0$, $x = 0$ e $y + z = 9$. A densidade é dada por $\delta(x, y, z) = x + y$.

Solução



$$W: \begin{cases} 0 \leq y \leq 9 \\ 0 \leq x \leq \sqrt{y} \\ 0 \leq z \leq 9 - y \end{cases}$$

ou também:

$$W: \begin{cases} 0 \leq x \leq 3 \\ x^2 \leq y \leq 9 \\ 0 \leq z \leq 9 - y \end{cases}$$

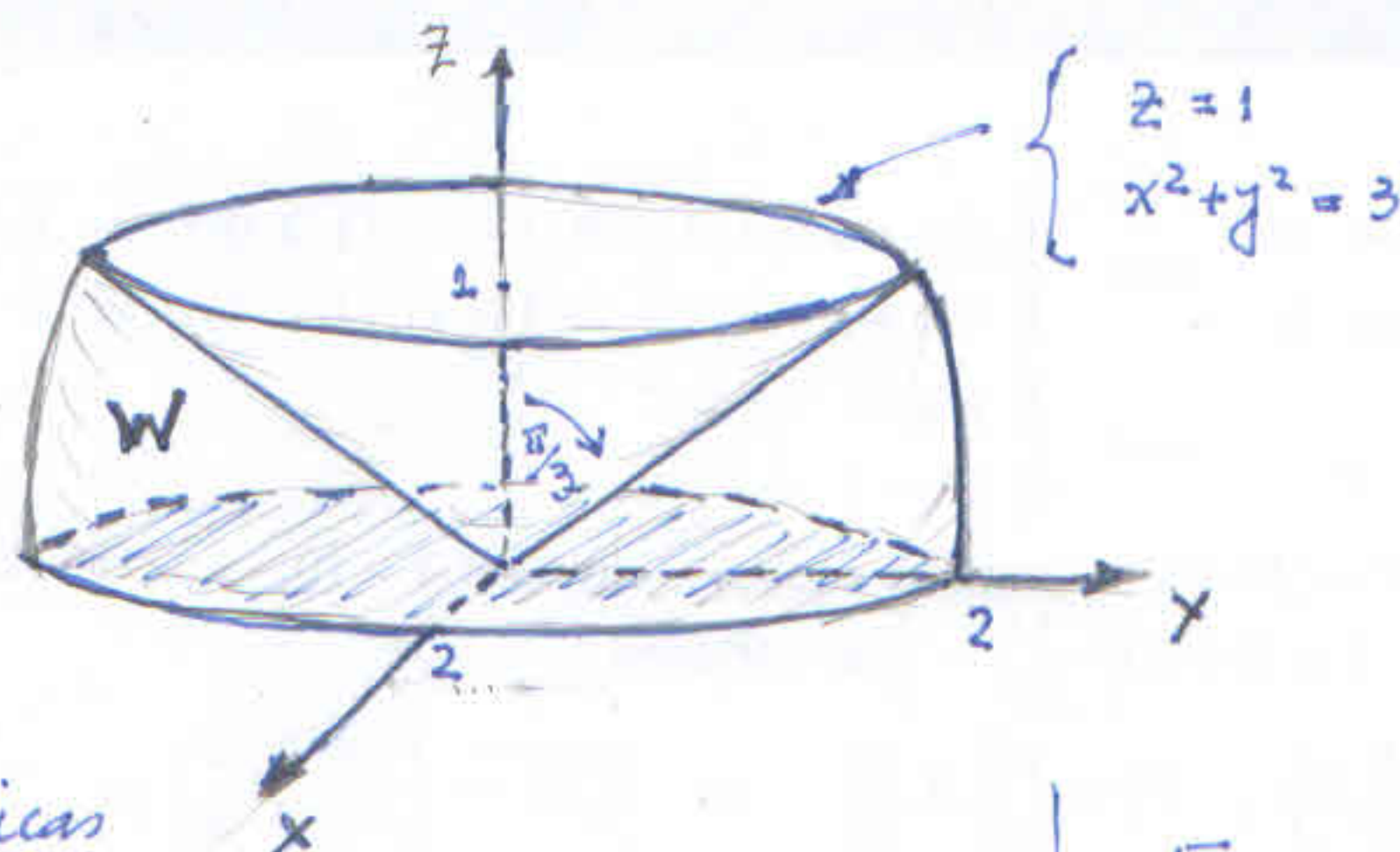
$$\text{Massa de } W = M(W) = \int_0^3 \int_{x^2}^9 \int_0^{9-y} (x+y) dz dy dx$$

$$= \int_0^3 \int_{x^2}^9 (x+y)(9-y) dy dx = \int_0^3 \left(9xy + \frac{9-x}{2} y^2 - \frac{y^3}{3} \right) \Big|_{x^2}^9 dx$$

$$= \dots = \frac{53.703}{140}$$

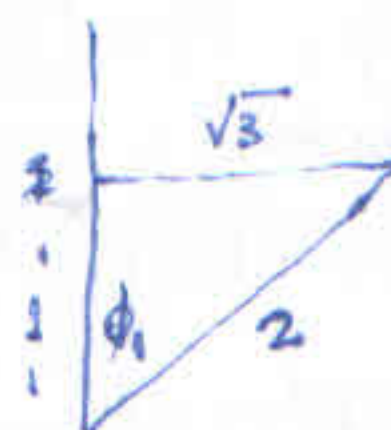
- ⑨ Calcule o volume do sólido acima do plano $z=0$, dentro da esfera $x^2+y^2+z^2=4$ e abaixo do cone $z=\sqrt{\frac{x^2+y^2}{3}}$.

Solução



W em coord. esféricas

$$W: \begin{cases} 0 \leq \theta \leq 2\pi \\ 0 \leq \rho \leq 2 \\ \pi/3 \leq \phi \leq \pi/2 \end{cases}$$



$$\tan \phi_1 = \sqrt{3}$$

$$\sec \phi_1 = \sqrt{3}/2$$

$$\cos \phi_1 = 1/2$$

$$\phi_1 = \pi/3$$

$$\text{vol}(W) = \int_0^{2\pi} \int_0^2 \int_{\pi/3}^{\pi/2} \rho^2 \sin \phi \, d\phi \, d\rho \, d\theta$$

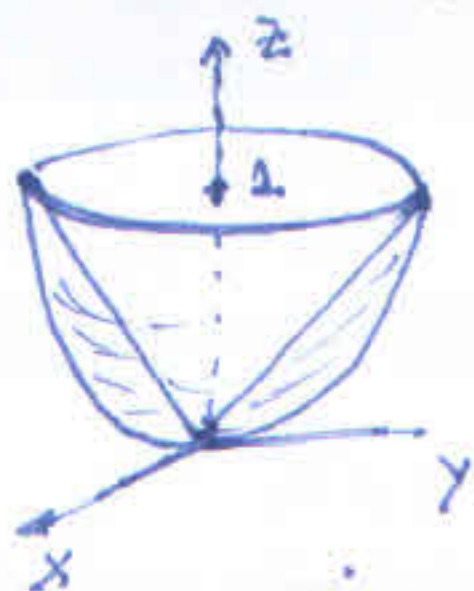
$$= \int_0^{2\pi} \int_0^2 \left[-\rho^2 \cos \phi \right]_{\pi/3}^{\pi/2} d\rho \, d\theta = \int_0^{2\pi} \int_0^2 -\rho^2 (0 - 1/2) d\rho \, d\theta$$

$$= \int_0^{2\pi} \int_0^2 \frac{1}{2} \rho^2 d\rho \, d\theta = \frac{\pi}{3} \left[\rho^3 \right]_0^2 = \frac{8\pi}{3} \text{ u.v.}$$

(10)

Calcule o volume do sólido ^W acima do paraboloide $z = x^2 + y^2$ e abaixo do cone $z = \sqrt{x^2 + y^2}$

Solução:



$$W = \left\{ \begin{array}{l} x^2 + y^2 \leq 1 \\ x^2 + y^2 \leq z \leq \sqrt{x^2 + y^2} \end{array} \right.$$

$$\text{vol}(W) = \iiint_W dV$$

W em coord. cilíndricas

$$W : \left\{ \begin{array}{l} 0 \leq \theta \leq 2\pi \\ 0 \leq r \leq 1 \\ r^2 \leq z \leq r \end{array} \right.$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$z = z$$

$$\underline{dx dy dz = dV = r dr d\theta dz}$$

$$\text{vol}(W) = \int_0^1 \int_{r^2}^r \int_0^{2\pi} r d\theta dz dr = \int_0^1 \int_{r^2}^r 2\pi r dz dr$$

$$= \int_0^1 2\pi r (r - r^2) dr = 2\pi \int_0^1 (r^2 - r^3) dr$$

$$= 2\pi \left(\frac{r^3}{3} - \frac{r^4}{4} \right) \Big|_0^1 = 2\pi \left(\frac{1}{3} - \frac{1}{4} \right)$$

$$= \underline{\underline{\pi/6}} \quad \text{u.v.}$$