

$$\textcircled{1} \quad W: \begin{cases} 0 \leq z \leq h & h = \text{cte.} > 0 \\ a \leq x^2 + y^2 \leq b, & b > a > 0 \end{cases}$$

$$I_z = \iiint_W (x^2 + y^2) \cdot K \, dx \, dy \, dz$$



Em coord. cilíndricas W é descrito como:

$$W: \begin{cases} \sqrt{a} \leq r \leq \sqrt{b} \\ 0 \leq \theta \leq 2\pi \\ 0 \leq z \leq h. \end{cases}$$

$$\underline{dx \, dy \, dz = r \, dr \, d\theta \, dz}$$

$$I_z = \int_0^h \int_0^{2\pi} \int_{\sqrt{a}}^{\sqrt{b}} K \cdot r^2 \cdot r \, dr \, d\theta \, dz$$

$$= \int_0^h \int_0^{2\pi} K \left. \frac{r^4}{4} \right|_{\sqrt{a}}^{\sqrt{b}} d\theta \, dz = \frac{K}{4} (b^2 - a^2) \int_0^h \int_0^{2\pi} d\theta \, dz$$

$$= \frac{K h}{4} \cdot 2\pi (b^2 - a^2)$$

$$= \frac{K h}{2} \pi (b^2 - a^2)$$

②

$$(a) \iiint_W \frac{1}{z^2} dx dy dz$$

$$W: z = \sqrt{x^2 + y^2}, \quad z = \sqrt{1 - x^2 - y^2}, \quad z = \sqrt{4 - x^2 - y^2}$$



Em coord esféricas W é dado por

$$W: \begin{cases} 0 \leq \theta \leq 2\pi \\ 1 \leq \rho \leq 2 \\ 0 \leq \phi \leq \pi/4 \end{cases}$$

$$\begin{cases} x = \rho \cos \theta \sin \phi \\ y = \rho \sin \theta \sin \phi \\ z = \rho \cos \phi \end{cases}$$

$$\iiint_W \frac{1}{z^2} dx dy dz = \int_0^{2\pi} \int_1^2 \int_0^{\pi/4} \frac{1}{\rho^2 \cos^2 \phi} \cdot \rho^2 \sin \phi d\phi d\rho d\theta$$

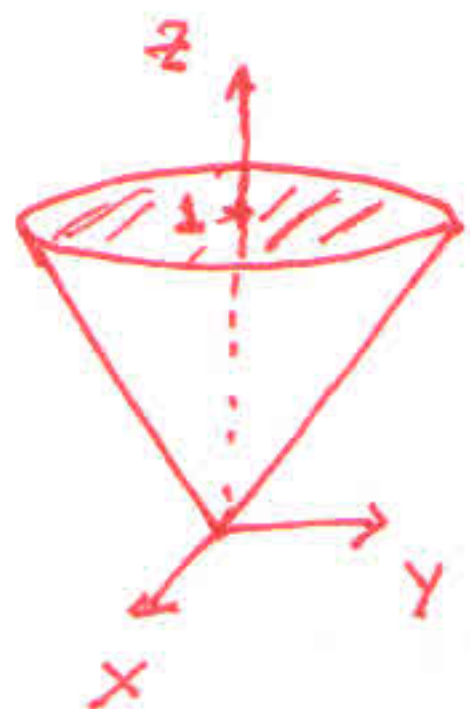
$$= \int_0^{2\pi} \int_1^2 \int_0^{\pi/4} \left(\frac{\sin \phi}{\cos \phi} \right) \left(\frac{1}{\cos \phi} \right) d\phi d\rho d\theta$$

$$= \int_0^{2\pi} \int_1^2 \int_0^{\pi/4} \sec \phi \cdot \tan \phi d\phi d\rho d\theta = \int_0^{2\pi} \int_1^2 \sec \phi \Big|_0^{\pi/4} d\rho d\theta$$

$$= \int_0^{2\pi} \int_1^2 (\sqrt{2} - 1) d\rho d\theta = 2\pi(\sqrt{2} - 1)$$

$$(2) (b) \iiint_W (1 + \sqrt{x^2 + y^2}) dx dy dz$$

$$W: \begin{cases} z = \sqrt{x^2 + y^2} \\ z = 1 \end{cases}$$



W em coord. cilíndricas:

$$W: \begin{cases} 0 \leq r \leq 1 \\ 0 \leq \theta \leq 2\pi \\ r \leq z \leq 1 \end{cases}$$

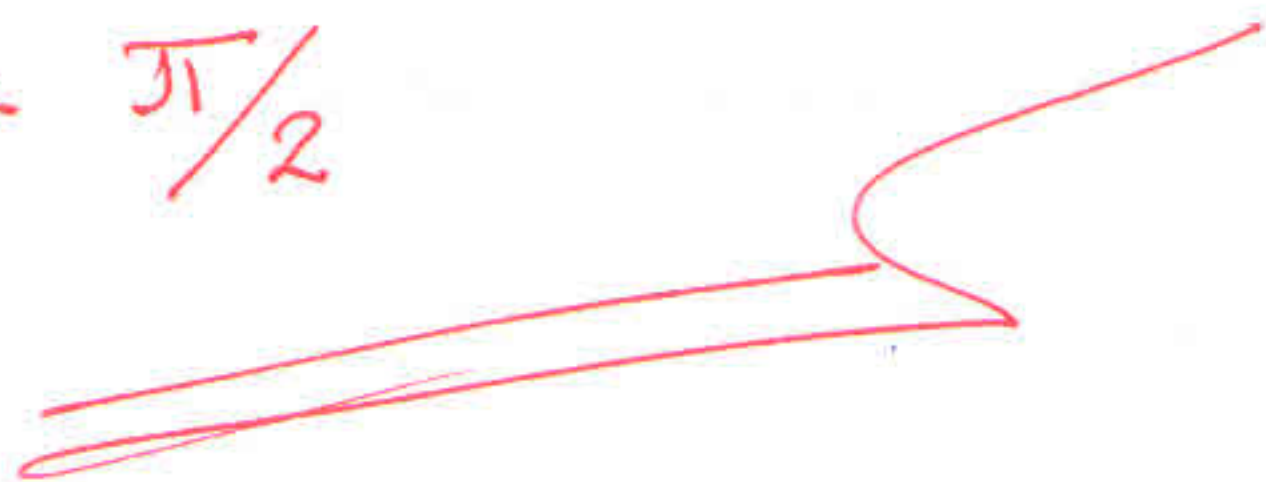
$$\begin{aligned} x &= r \cos \theta \\ y &= r \sin \theta \\ z &= z \\ x^2 + y^2 &= r^2 \end{aligned}$$

$$\iiint_W (1 + \sqrt{x^2 + y^2}) dx dy dz = \int_0^1 \int_0^{2\pi} \int_r^1 (1 + r) \cdot r dz d\theta dr$$

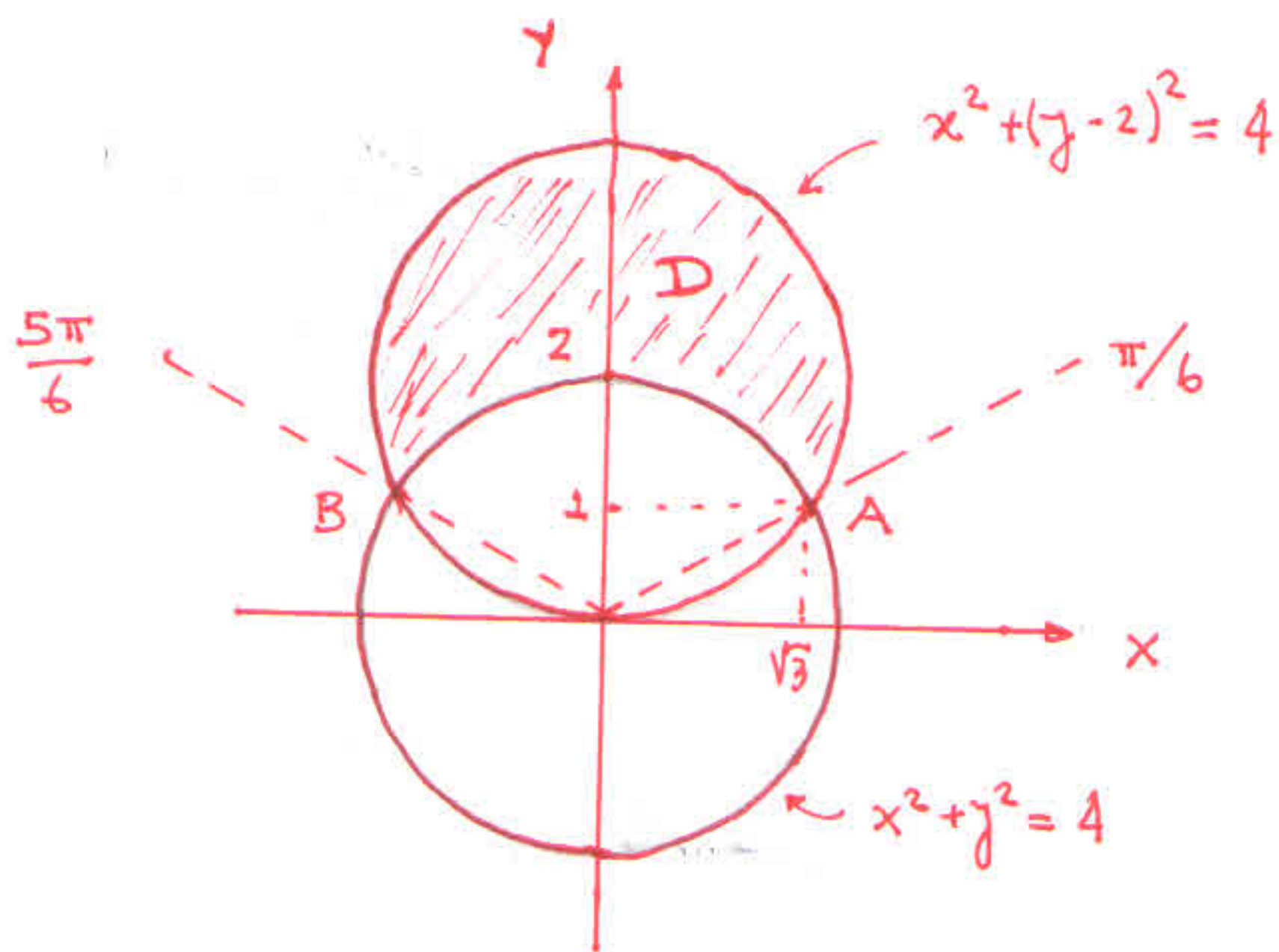
$$= \int_0^1 \int_0^{2\pi} (1 + r) \cdot r (1 - r) d\theta dr = 2\pi \int_0^1 (r - r^3) dr$$

$$= 2\pi \left(\frac{r^2}{2} - \frac{r^4}{4} \right) \Big|_0^1 = 2\pi \left(\frac{1}{2} - \frac{1}{4} \right)$$

$$= \pi/2$$



3



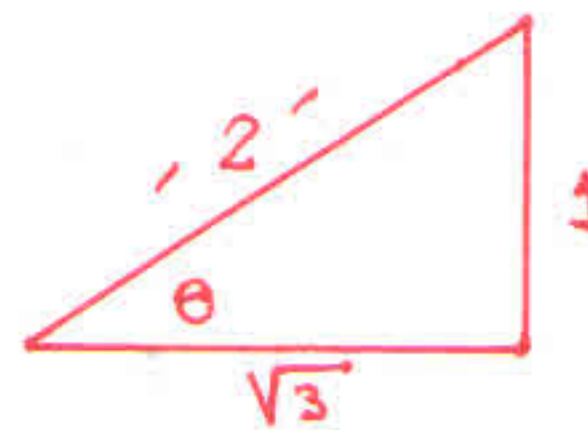
$$A = (\sqrt{3}, 1)$$

$$B = (-\sqrt{3}, 1)$$

~~$x^2 + y^2 = 4$~~

$$\left. \begin{array}{l} x^2 + y^2 = 4 \\ x^2 + (y-2)^2 = 4 \end{array} \right\} \Rightarrow y = 1$$

$$y = 1 \Rightarrow x = \pm \sqrt{3}$$



$$\operatorname{tg} \theta = \frac{1}{\sqrt{3}}$$

$$\operatorname{sen} \theta = \frac{1}{2}$$

$$\cos \theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30^\circ = \pi/6$$

$$\delta(x, y) = \sqrt{x^2 + y^2}$$

$$M = \text{Massa} = \iint_D \delta(x, y) dx dy = \iint_D \sqrt{x^2 + y^2} dx dy$$

Em coord. polares

$$x = r \cos \theta$$

$$y = r \operatorname{sen} \theta$$

$$dx dy = r dr d\theta$$

$$\left\{ \begin{array}{l} x^2 + y^2 = 4 \Rightarrow r = 2 \\ x^2 + (y-2)^2 = 4 \Rightarrow r = 4 \operatorname{sen} \theta \end{array} \right.$$

$$$$

D em coord. polares:

$$D: \left\{ \begin{array}{l} \frac{\pi}{6} \leq \theta \leq \frac{5\pi}{6} \\ 2 \leq r \leq 4 \operatorname{sen} \theta \end{array} \right.$$

$$M = \int_{\pi/6}^{5\pi/6} \int_2^{4 \operatorname{sen} \theta} r \cdot r dr d\theta = \int_{\pi/6}^{5\pi/6} \left. \frac{r^3}{3} \right|_2^{4 \operatorname{sen} \theta} d\theta$$

$$M = \int_{\pi/6}^{5\pi/6} \left(4 \frac{\sin^3 \theta}{3} - \frac{2^3}{3} \right) d\theta$$

$$= -\frac{8}{3} \cdot \frac{4\pi}{3} + \frac{4^3}{3} \int_{\pi/6}^{5\pi/6} \sin^3 \theta d\theta$$

$$= -\frac{16}{9} \pi + \frac{4^3}{3} \int_{\pi/6}^{5\pi/6} \sin^3 \theta d\theta$$

$$\begin{aligned} \int \sin^3 \theta d\theta &= \int \sin \theta (1 - \cos^2 \theta) d\theta = \int \sin \theta d\theta + \int \cos^2 \theta (-\sin \theta) d\theta \\ &= -\cos \theta + \frac{1}{3} \cos^3 \theta + C \end{aligned}$$

$$M = -\frac{16}{9} \pi + \frac{4^3}{3} \left(-\cos \theta \Big|_{\pi/6}^{5\pi/6} + \frac{1}{3} \cos^3 \theta \Big|_{\pi/6}^{5\pi/6} \right)$$

$$\left\{ \cos \frac{5\pi}{6} = -\cos \frac{\pi}{6} \right\}$$

$$M = -\frac{16}{9} \pi + \frac{4^3}{3} \cdot 2 \cos \frac{\pi}{6} - \frac{4^3}{3} \cdot \frac{2}{3} \cos^3 \frac{\pi}{6}$$

$$\left(\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \right)$$

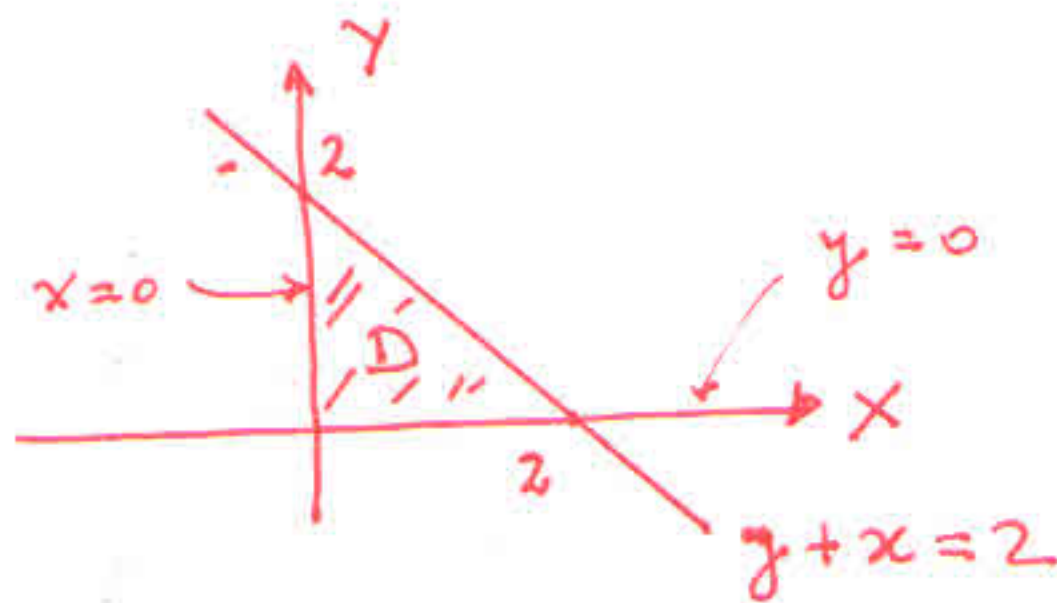
$$M = -\frac{16}{9} \pi + \frac{4^3}{3} \cdot 2 \cdot \frac{\sqrt{3}}{2} - \frac{4^3}{3} \cdot \frac{2}{3} \left(\frac{\sqrt{3}}{2} \right)^3$$

$$= -\frac{16}{9} \pi + \frac{56}{3} \sqrt{3}$$

④

$$\iint_D e^{\frac{y-x}{x+y}} dx dy$$

D:



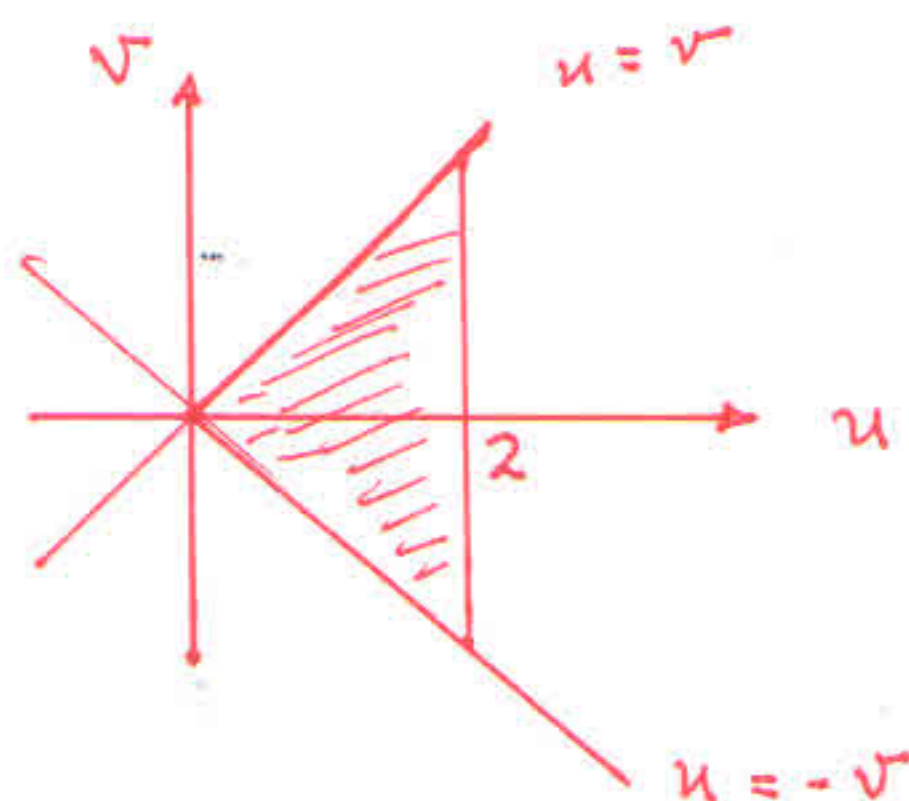
Faz

$$\begin{cases} u = x+y \\ v = y-x \end{cases}$$

$$x=0 \Rightarrow u=v$$

$$y=0 \Rightarrow u=-v$$

$$y+x=2 \Rightarrow u=2$$



$$\iint_D e^{\frac{y-x}{x+y}} dx dy = \int_0^2 \int_{-u}^u e^{v/u} \cdot |J| dv du$$

$$J^{-1} = \det \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} = 2 \Rightarrow \underline{J = 1/2}$$

$$\int_0^2 \int_{-u}^u \frac{1}{2} e^{v/u} dv du = \frac{1}{2} \int_0^2 u e^{v/u} \Big|_{v=-u}^{v=u} du$$

$$= \frac{1}{2} \int_0^2 u (e - e^{-1}) du = \frac{1}{2} (e - \frac{1}{e}) \int_0^2 u du$$

$$= \frac{1}{2} (e - \frac{1}{e}) \frac{u^2}{2} \Big|_0^2$$

$$= e - \frac{1}{e}$$

