Lista 4

(1)

(a) $\vec{F}(x,y) = (2xy^2 - y^3)\vec{i} + (2x^2y - 3xy^2 + 2)\vec{j}$

$P = 2xy^2 - y^3 \quad , \quad Q = 2x^2y - 3xy^2 + 2$

Queremos encontrar $\varphi = \varphi(x,y)$ t.q. $\frac{\partial \varphi}{\partial x} = P$ e $\frac{\partial \varphi}{\partial y} = Q$

$\frac{\partial \varphi}{\partial x} = P = 2xy^2 - y^3 \Rightarrow \varphi = x^2y^2 - xy^3 + f(y)$

$\Rightarrow \frac{\partial \varphi}{\partial y} = 2x^2y - 3xy^2 + f'(y)$

Mas $\frac{\partial \varphi}{\partial y} = Q = 2x^2y - 3xy^2 + 2$, logo

$\cancel{2x^2} - \cancel{3xy^2} + f'(y) = \cancel{2x^2y} - \cancel{3xy^2} + 2$

daí $f'(y) = 2$, $\therefore$ $f(y) = 2y + \text{cte.}$

Logo $\boxed{\varphi = x^2y^2 - xy^3 + 2y + C}$

(b) $\vec{F}(x,y,z) = 2xy\,\vec{i} + (x^2 + z\cos yz)\vec{j} + y\cos yz\,\vec{k}$

$P = 2xy \quad , \quad Q = x^2 + z\cos yz \quad , \quad R = y\cos yz$

Queremos $\varphi = \varphi(x,y,z)$ t.q. $\frac{\partial \varphi}{\partial x} = P$, $\frac{\partial \varphi}{\partial y} = Q$ e $\frac{\partial \varphi}{\partial z} = R$.

$\frac{\partial \varphi}{\partial x} = P \Rightarrow \varphi = x^2y + f(y,z)$

$\frac{\partial \varphi}{\partial y} = Q \Rightarrow \varphi = x^2y + \operatorname{sen} yz + g(x,z)$

$\frac{\partial \varphi}{\partial z} = R \Rightarrow \varphi = \operatorname{sen} yz + h(x,y)$

Daí obtemos,

$$f(y,z) = \text{sen}\, yz \quad , \quad g(x,z) = 0 \quad e \quad h(x,y) = x^2 y .$$

Logo $\quad \varphi(x,y,z) = x^2 y + \text{sen}\, yz \quad$ ou também

$$\varphi(x,y,z) = x^2 y + \text{sen}\, yz + C \quad , \quad C = \text{cte.}$$

(c) $\quad \vec{F}(x,y) = (e^{x+y} + 1)\,\vec{i} + e^{x+y}\,\vec{j}$

$$P = e^{x+y} + 1 \quad , \quad Q = e^{x+y}$$

$$\frac{\partial \varphi}{\partial x} = P = e^{x+y} + 1 \implies \varphi = e^{x+y} + x + f(y)$$

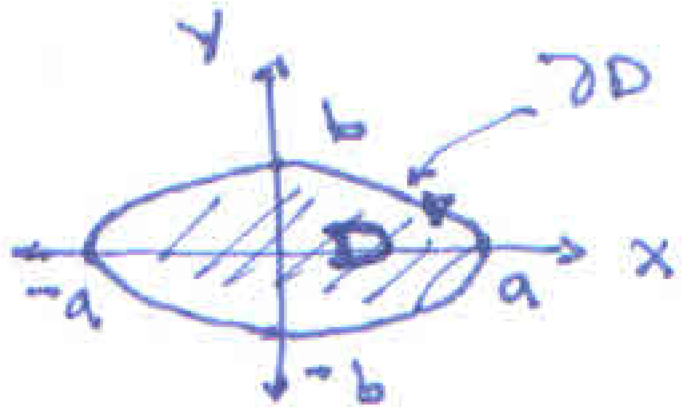
$$\frac{\partial \varphi}{\partial y} = Q = e^{x+y} \implies \varphi = e^{x+y} + g(x)$$

Daí, $\quad f(y) = 0 \quad e \quad g(x) = x .$

$$\therefore \quad \varphi(x,y) = e^{x+y} + x \quad (+\ \text{cte})$$

3) Use o teor. de Green p/ calcular a área.

(a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (elipse)



$$\text{área}(D) = \iint_D dxdy = \iint_D (0+1)dxdy = \iint_D \left(\frac{\partial 0}{\partial x} - \frac{\partial(-y)}{\partial y}\right)dxdy$$

$$\overset{\text{Green}}{=} \int_{\partial D^+} -y\,dx + 0\,dy = \int_{\partial D^+} -y\,dx$$

∂D é parametrizada por $\gamma(t) = (a\cos t, b\,\text{sen}\,t)$, $0 \le t \le 2\pi$.

$\gamma'(t) = (-a\,\text{sen}\,t, b\cos t)$

$$\therefore \quad \text{área}(D) = \int_0^{2\pi} (-b\,\text{sen}\,t)\cdot(-a\,\text{sen}\,t)\,dt = \int_0^{2\pi} ab\,\text{sen}^2 t\,dt$$

$$= ab\,\frac{1}{2}\left(t - \frac{\text{sen}\,2t}{2}\right)\Big|_0^{2\pi} = ab\cdot\frac{1}{2}\cdot 2\pi$$

$$= \pi ab \quad \text{u.a.}$$

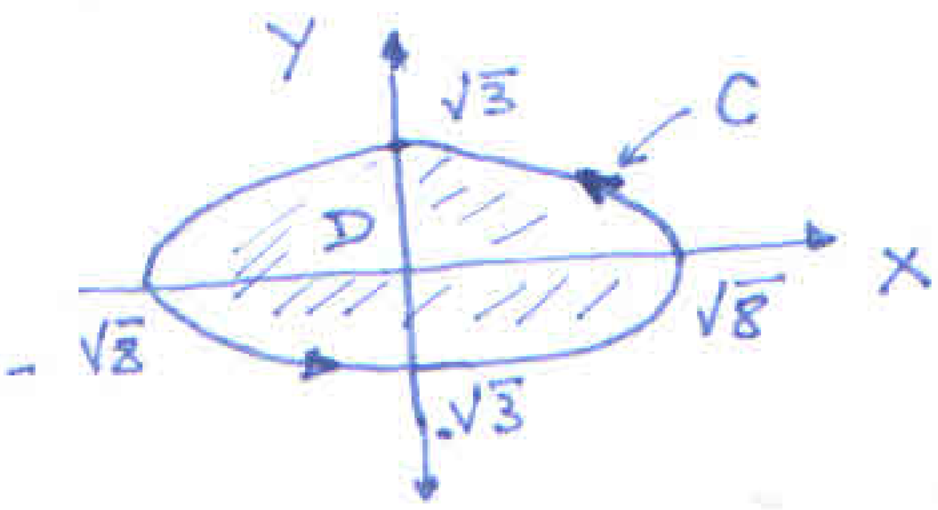
(4) (a) Calcular usando Green

$$\oint_C e^x \operatorname{sen} y \, dx + (x + e^x \cos y)\, dy$$, onde C é a elipse

$3x^2 + 8y^2 = 24$ orientada no sentido anti-horário.

Solução:

$$3x^2 + 8y^2 = 24 \Rightarrow \frac{x^2}{8} + \frac{y^2}{3} = 1$$



D = região limitada por C
$(C = \partial D)$

$$\vec{F}(x,y) = P\vec{i} + Q\vec{j} = e^x \operatorname{sen} y \, \vec{i} + (x + e^x \cos y)\vec{j}$$

Note que $\vec{F}$ é de classe C^1 em $\mathbb{R}^2$.

Pelo teor. de Green, $\int_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dx\,dy$.

$$\therefore \oint_C e^x \operatorname{sen} y \, dx + (x + e^x \cos y)\, dy = \iint_D \left(1 + e^x \cos y - e^x \cos y\right) dx\,dy$$

$$= \iint_D dx\,dy = \text{Área}(D)$$

$$= \sqrt{3}\sqrt{8}\,\pi = 2\sqrt{6}\,\pi$$