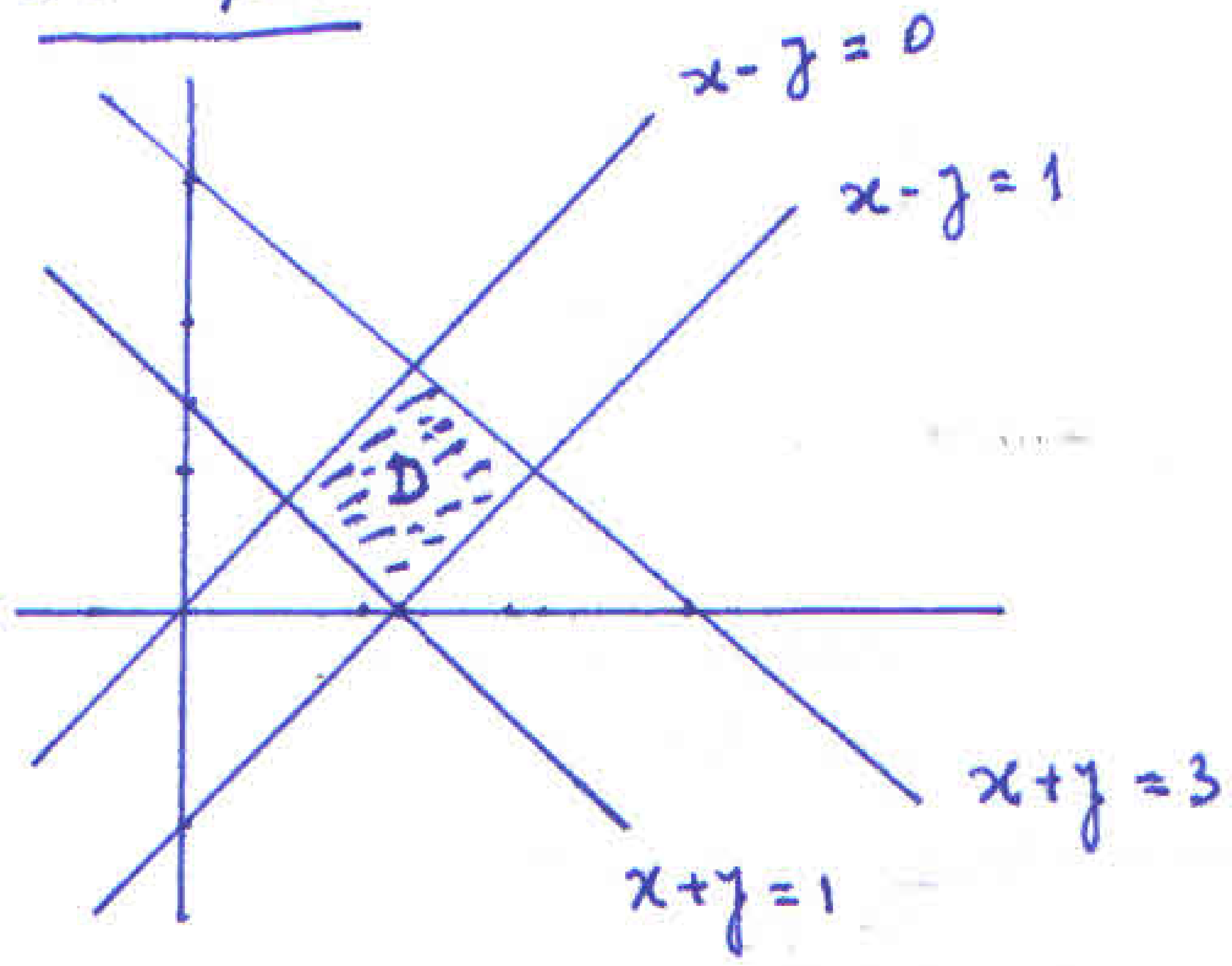
# LISTA 1

(1) $\displaystyle\iint_D \frac{x-y}{x+y}\, dA$ $\quad$ D é a região limitada pelas retas $x-y=0$, $x-y=1$, $x+y=1$ e $x+y=3$

Solução:




Note a ocorrência $x-y$ e $x+y$ no integrando e nas eqs das retas que limitam a região D

.... ... ...

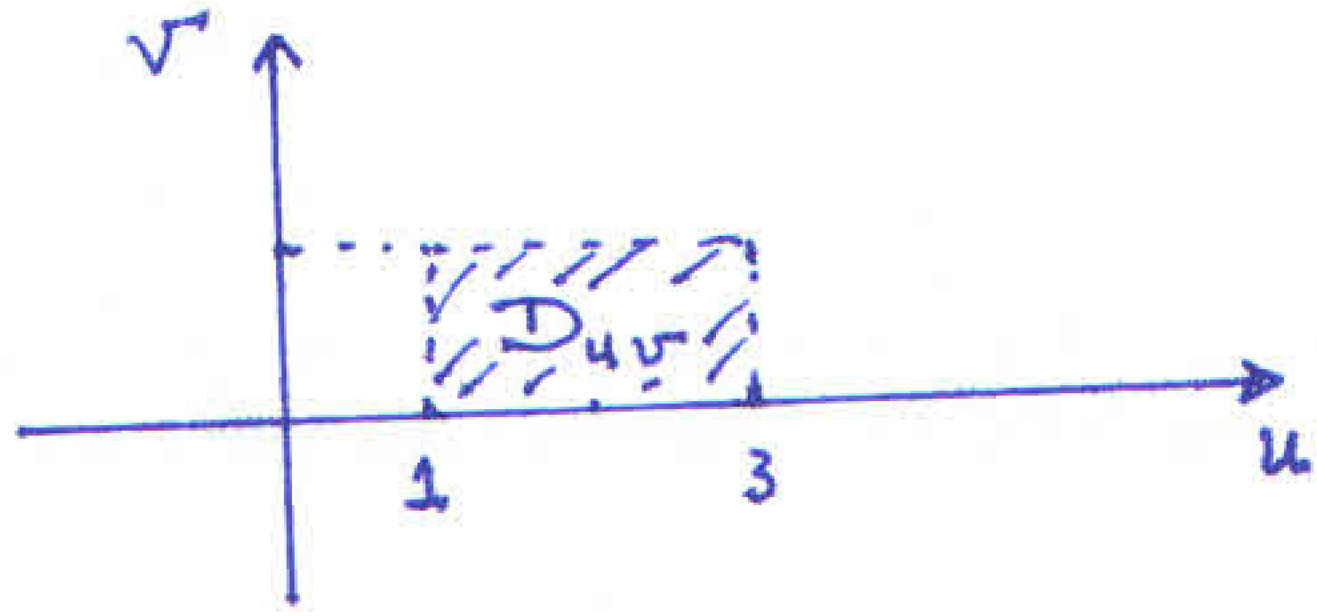
Isto sugere a mudança de variáveis $\begin{cases} u = x+y \\ v = x-y \end{cases}$

Então, $\quad x = \dfrac{u+v}{2}$, $y = \dfrac{u-v}{2}$.

O Jacobiano é

$$J = \det\begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix} = \det\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & -1/2 \end{pmatrix} = -\frac{1}{4} - \frac{1}{4} = -1/2$$

Com essa mudança de var. $D_{uv}$ agora é limitado pelas retas $v=0$, $v=1$, $u=1$ e $u=3$




Assim, $\displaystyle\iint_D \frac{x-y}{x+y}\, dA = \iint_{D_{uv}} \frac{v}{u}\, |J|\, du\, dv = \iint_{D_{uv}} \frac{v}{u}\, |-1/2|\, du\, d$

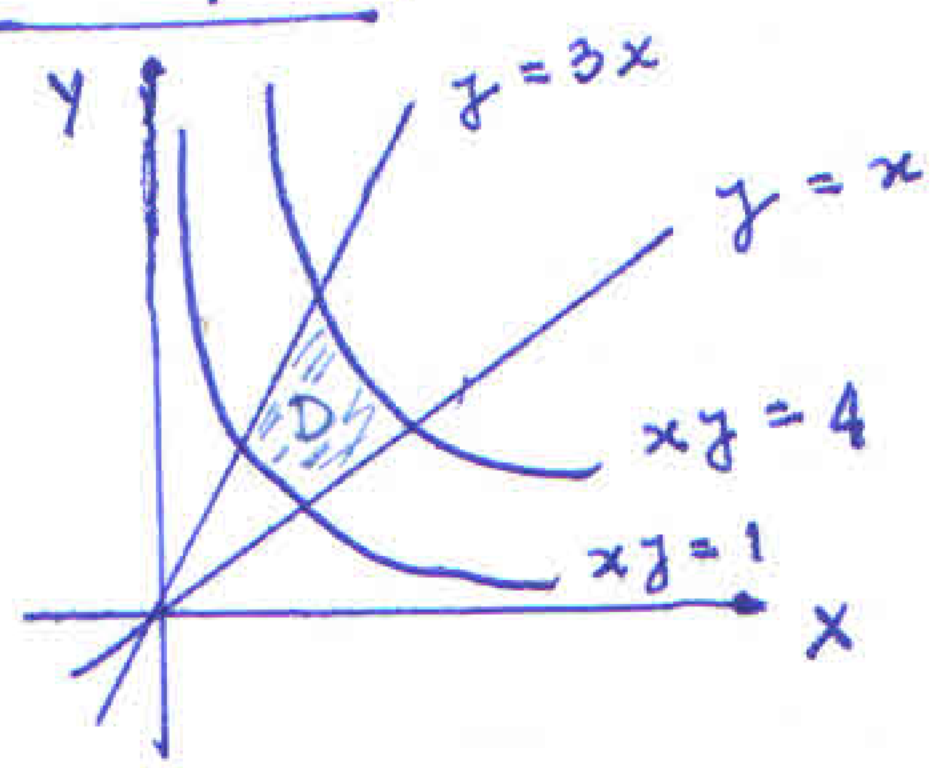
$$\iint_D \frac{x-y}{x+y}\,dA = \frac{1}{2}\int_0^1\int_1^3 \frac{v}{u}\,du\,dv$$

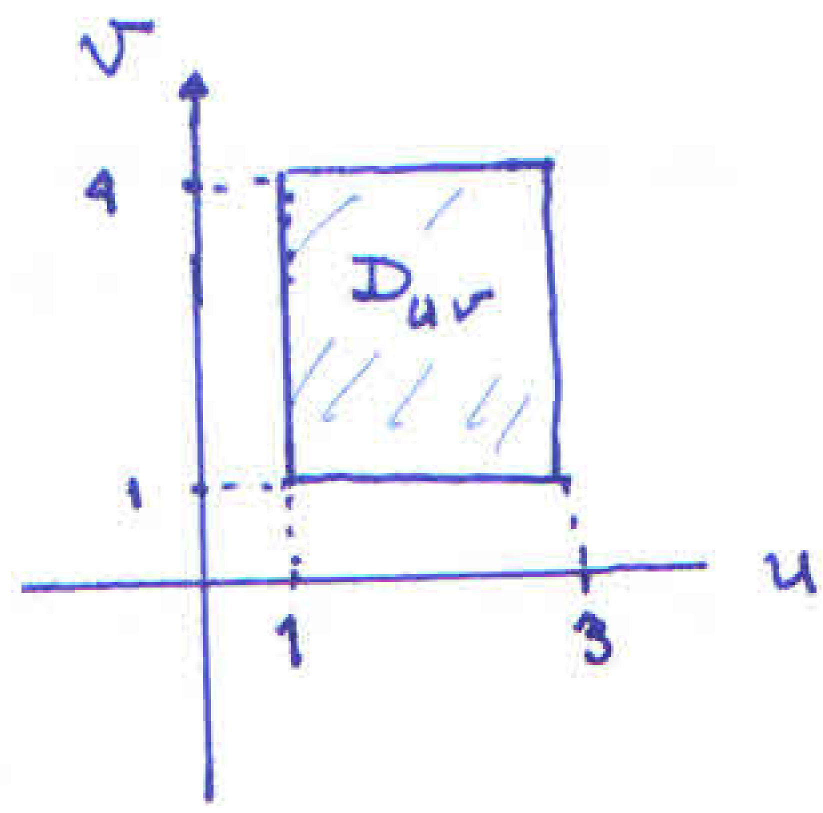
$$= \dots \quad \dots = \underline{\underline{\frac{1}{4}\ln 3}}$$

(2) $D$ é a região do 1º quadrante limitada por $y = x$, $y = 3x$, $xy = 1$ e $xy = 4$

Calcule $\displaystyle\iint_D xy^3\,dA$ $\left(\text{use: } \begin{cases} u = y/x \\ v = xy \end{cases}\right)$

<u>Solução</u>:




$$J^{-1} = \begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix} = \begin{vmatrix} -y/x^2 & 1/x \\ y & x \end{vmatrix} = -\frac{y}{x} - \frac{y}{x} = -\frac{2y}{x} = -2u$$

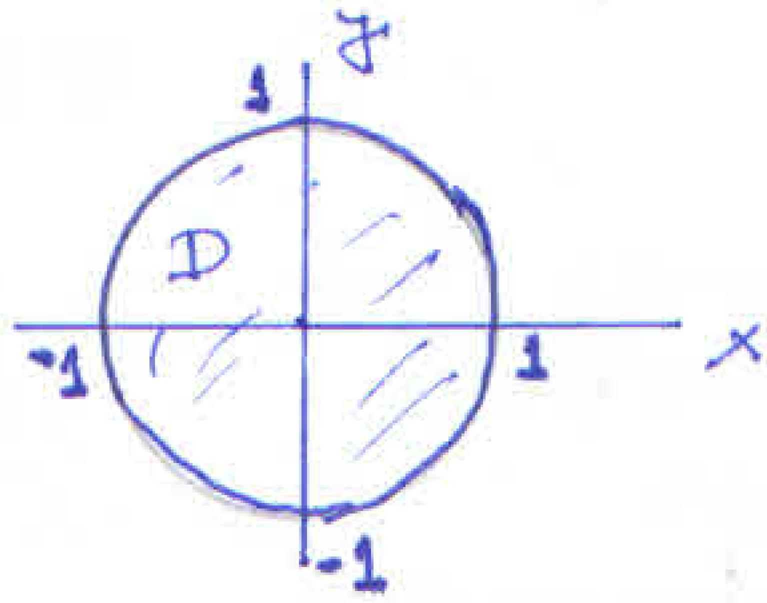
$$J^{-1} = -2u \Rightarrow \underline{J = \frac{-1}{2u}}$$

$$u = y/x\,,\ v = xy \Rightarrow \underline{u \cdot v = y^2}$$

$$\iint_D xy^3\,dA = \int_{D_{uv}} u v^2 \left|\frac{-1}{2u}\right| du\,dv = \frac{1}{2}\int_1^3\int_1^4 v^2\,dv\,du$$

$$= \dots \quad = \underline{\underline{21}}$$

3 $\iint_D e^{-(x^2+y^2)}\,dA \qquad D: \; x^2+y^2 \le 1$




$$\begin{cases} x = r\cos\theta \\ y = r\,\text{sen}\,\theta \end{cases}$$

$$dA = r\,dr\,d\theta$$

$$x^2+y^2 = r^2$$

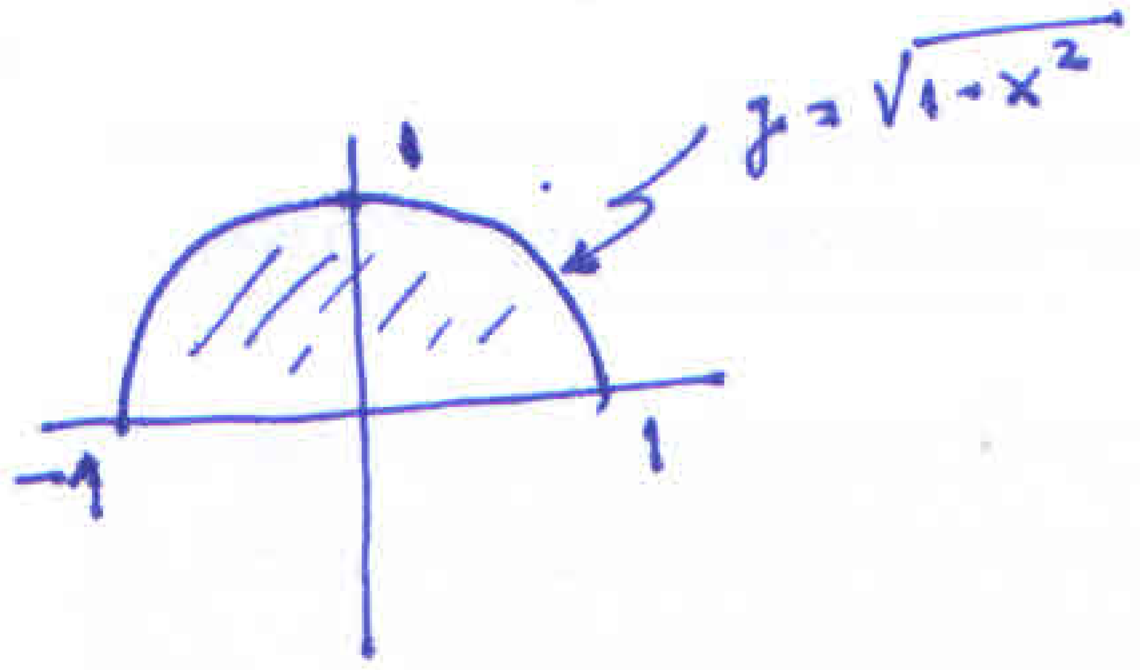
$$\iint_D e^{-(x^2+y^2)}\,dA = \int_0^1 \int_0^{2\pi} e^{-r^2}\, r\,d\theta\,dr$$

$$= \int_0^1 2\pi r e^{-r^2}\,dr = 2\pi \int_0^1 r e^{-r^2}\,dr = -\pi e^{-r^2}\Big|_0^1$$

$$= -\pi\left(e^{-1} - e^{0}\right) = -2\pi\left(\frac{1}{e} - 1\right) = 2\pi\left(1 - \frac{1}{e}\right)$$

**4.(a)**

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} (x^2+y^2)^{3/2}\,dy\,dx$$




$x = r\cos\theta$

$y = r\,\mathrm{sen}\,\theta$

$x^2+y^2=r^2$

$dx\,dy = r\,dr\,d\theta$

$-1 \leq x \leq 1$

$0 \leq y \leq \sqrt{1-x^2}$

$0 \leq r \leq 1$

$0 \leq \theta \leq \pi$

$$\int_{-1}^{1}\int_{0}^{\sqrt{1-x^2}} (x^2+y^2)^{3/2}\,dy\,dx = \int_{0}^{1}\int_{0}^{\pi} (r^2)^{3/2}\cdot r\,d\theta\,dr$$

$$= \int_{0}^{1}\int_{0}^{\pi} r^4\,d\theta\,dr = \pi\int_{0}^{1} r^4\,dr = \underline{\underline{\pi/5}}$$

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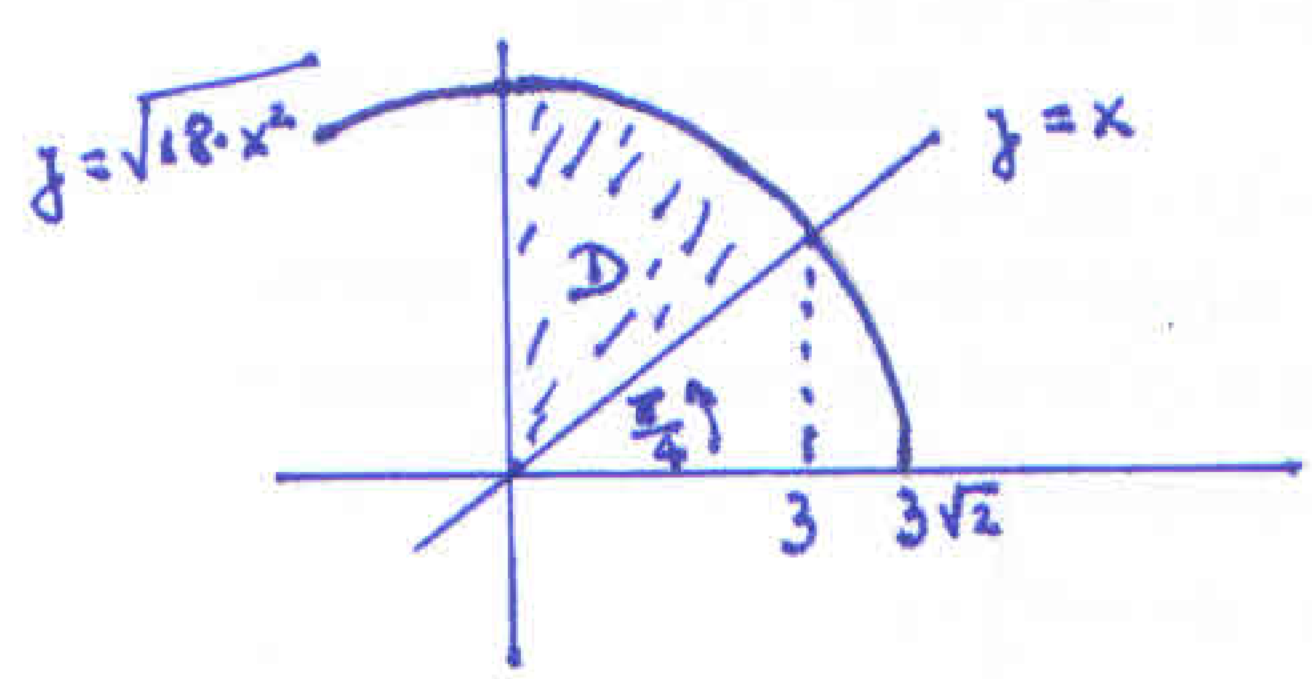
**4.(b)**

$$\int_{0}^{3}\int_{x}^{\sqrt{18-x^2}} (x^2+y^2+1)\,dy\,dx$$




$x = r\cos\theta$

$y = r\,\mathrm{sen}\,\theta$

$x^2+y^2=r^2$

$dx\,dy = r\,dr\,d\theta$

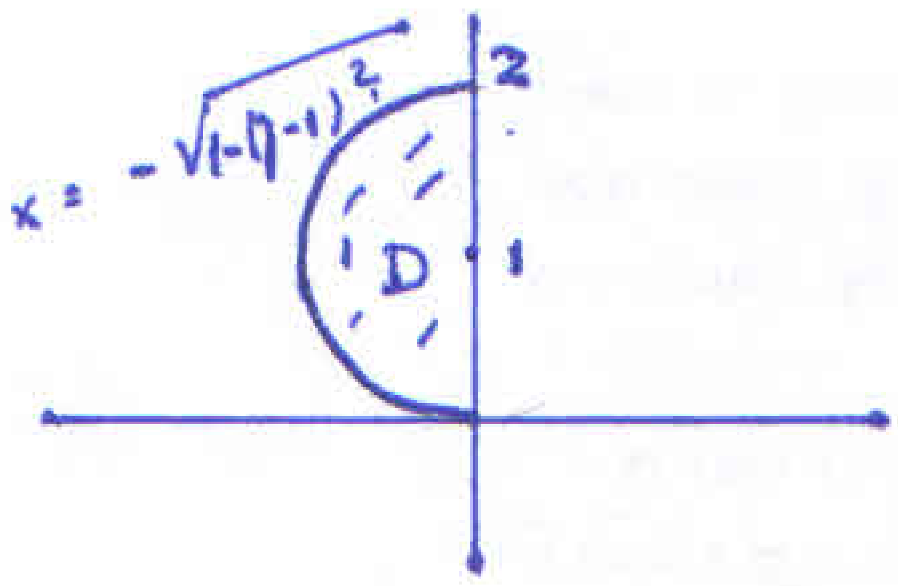
$\pi/4 \leq \theta \leq \pi/2$

$0 \leq r \leq 3\sqrt{2}$

$$\int_{0}^{3}\int_{x}^{\sqrt{18-x^2}} (x^2+y^2+1)\,dy\,dx = \int_{0}^{3\sqrt{2}}\int_{\pi/4}^{\pi/2} (r^2+1)\,r\,d\theta\,dr = \frac{\pi}{4}\int_{0}^{3\sqrt{2}} (r^3+r)\,dr$$

$$= \cdots = \frac{90\pi}{4} = \underline{\underline{\frac{45\pi}{2}}}$$

4.(a) $$\int_0^2 \int_{-\sqrt{1-(y-1)^2}}^{0} x y^2 \, dx\, dy$$




$\frac{\pi}{2} \leq \theta \leq \pi$

$\cancel{0 \leq r \leq -2\cos\theta}$

$0 \leq r \leq 2\,\text{sen}\,\theta$

$xy^2 = r\cos\theta \cdot r^2 \text{sen}^2\theta$

$$x^2 = 1-(y-1)^2$$
$$= -y^2+2y$$
$$x^2+y^2 = 2y$$
$$r^2 = 2r\,\text{sen}\,\theta$$
$$\boxed{r = 2\,\text{sen}\,\theta}$$

$$\int_0^2 \int_{-\sqrt{1-(y-1)^2}}^{0} x y^2 \, dx\, dy$$

$$\|$$

$$\int_{\pi/2}^{\pi} \int_0^{2\,\text{sen}\,\theta} r^3 \text{sen}^2\theta \cos\theta \cdot r\, dr\, d\theta$$

$$= \int_{+\pi/2}^{\pi} \frac{r^4}{4}\Big|_0^{2\,\text{sen}\,\theta} \text{sen}^2\theta \cdot \cos\theta \, d\theta = \int_{\pi/2}^{\pi} 4\,\text{sen}^6\theta \cdot \cos\theta\, d\theta$$

$$= \frac{4}{7}\,\text{sen}^7\theta \Big|_{\pi/2}^{\pi} = \frac{4}{7}(0-1) = -\frac{4}{7}$$

4.(d)

$$\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2}\, dy\, dx$$

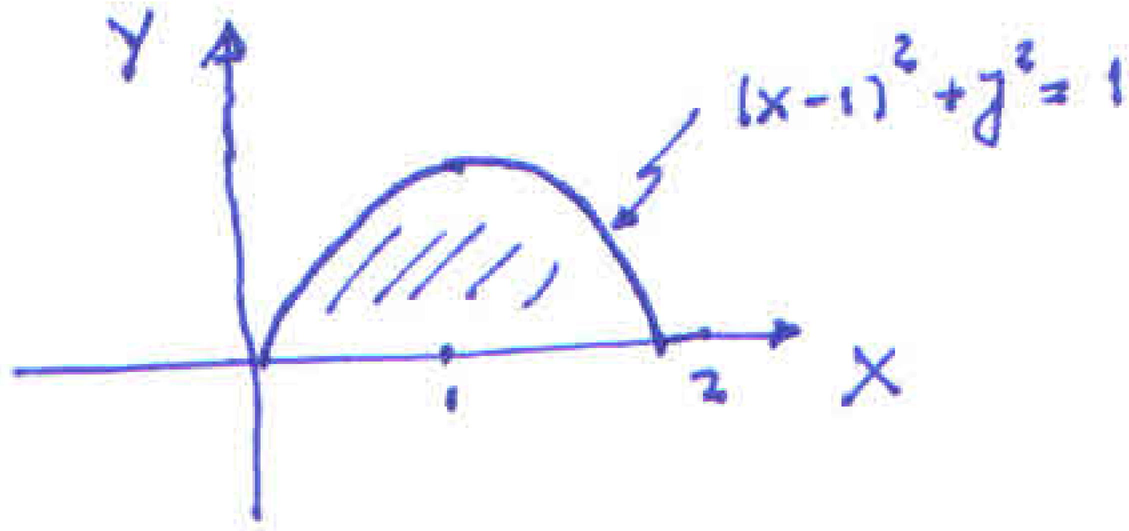
$x = r\cos\theta$

$y = r\,\text{sen}\,\theta$

$dy\,dx = r\,dr\,d\theta$

$x^2+y^2 = r^2$

$x+y = r(\cos\theta + \text{sen}\,\theta)$




$1 = (x-1)^2 + y^2 = x^2+y^2-2x+1$

$0 = r^2 - 2r\cos\theta$

$\underline{r = 2\cos\theta}$

$$\left\{ \begin{array}{l} 0 \le \theta \le \pi \\ 0 \le r \le 2\cos\theta \end{array} \right\}$$

$$\int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2}\, dy\, dx = \int_0^{\pi}\int_0^{2\cos\theta} \frac{r(\cos\theta+\text{sen}\,\theta)}{r^2}\cdot r\, dr\, d\theta$$

$$= \int_0^{\pi}\int_0^{2\cos\theta} (\cos\theta + \text{sen}\,\theta)\, dr\, d\theta = \int_0^{\pi} 2\cos\theta(\text{sen}\,\theta+\cos\theta)\, d\theta$$

$$= \int_0^{\pi} 2\,\text{sen}\,\theta\cos\theta\, d\theta + 2\int_0^{\pi}\cos^2\theta\, d\theta$$

$$\left( \begin{array}{l} \text{sen}\,2\theta = 2\,\text{sen}\,\theta\cos\theta \\ \cos 2\theta = \text{sen}^2\theta - \cos^2\theta \\ \cos 2\theta = -1 + 2\cos^2\theta \\ \cos^2\theta = \dfrac{1+\cos 2\theta}{2} \end{array} \right)$$

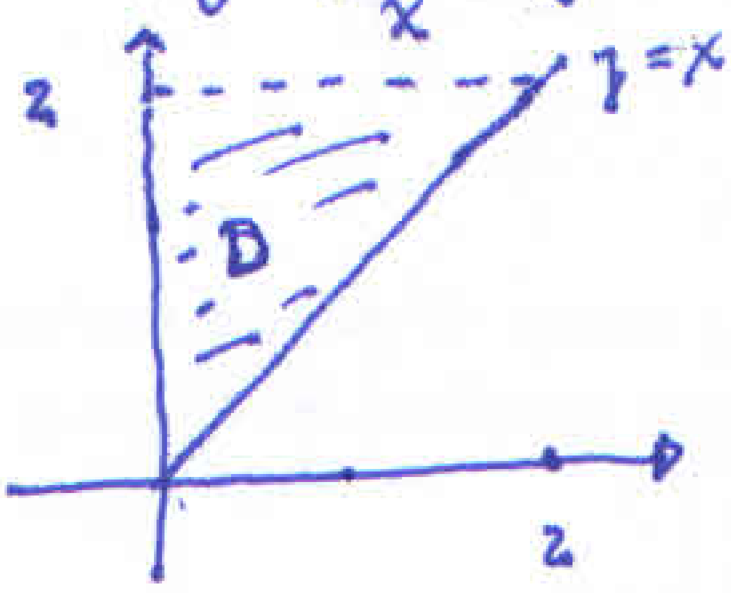
$$= \int_0^{\pi} \text{sen}\,2\theta\, d\theta + \int_0^{\pi} (1+\cos 2\theta)\, d\theta$$

$$= -\frac{1}{2}\cos 2\theta\Big|_0^{\pi} + \pi + \frac{1}{2}\text{sen}\,2\theta\Big|_0^{\pi}$$

$$= 0 + \pi - 0 = \pi$$

$$\therefore \left\{ \int_0^2 \int_0^{\sqrt{1-(x-1)^2}} \frac{x+y}{x^2+y^2}\, dy\, dx = \pi \right\}$$

5 (a) $\displaystyle\int_0^2 \int_x^2 2y^2 \operatorname{sen}(xy)\, dy\, dx$




$$\int_0^2 \int_x^2 2y^2 \operatorname{sen}(xy)\, dy\, dx = \int_0^2 \int_0^y 2y^2 \operatorname{sen}(xy)\, dx\, dy$$

$$= \int_0^2 2y^2 \cdot \frac{-1}{y} \cos(xy)\Big|_{x=0}^{x=y} dy = \int_0^2 -2y\left[\cos(y^2) - 1\right] dy$$

$$= -\int_0^2 2y \cos y^2\, dy + \int_0^2 2y\, dy = -\operatorname{sen} y^2 \Big|_0^2 + y^2\Big|_0^2$$

$$= -\operatorname{sen} 4 + 4 = \underline{\underline{4 - \operatorname{sen} 4}}$$

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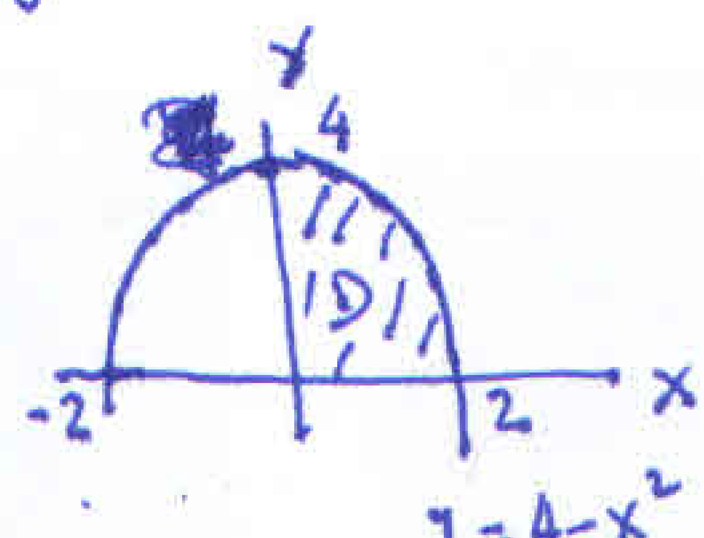
5. (b) $\displaystyle I = \int_0^2 \int_0^{4-x^2} \frac{x e^{2y}}{4-y}\, dy\, dx$



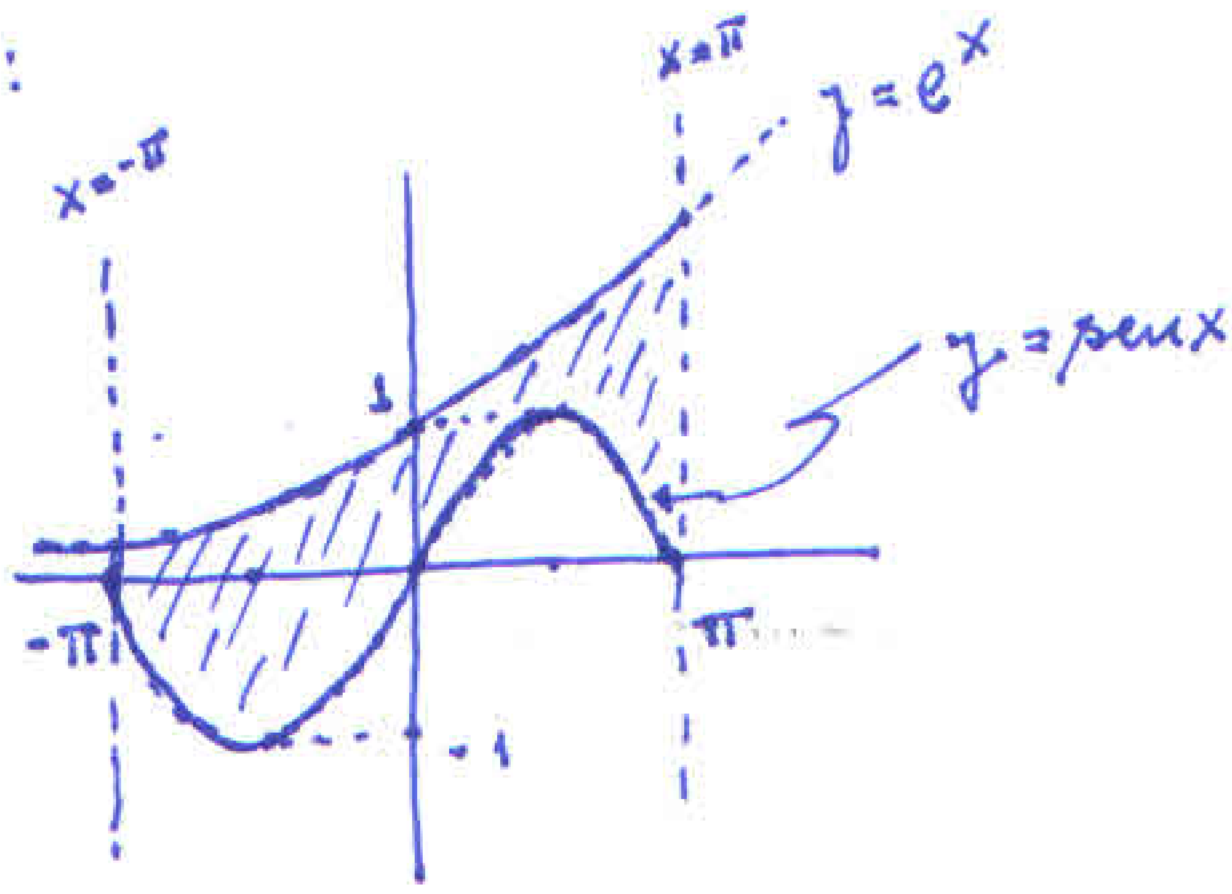

$$I = \int_0^4 \int_0^{\sqrt{4-y}} \frac{x e^{2y}}{4-y}\, dx\, dy = \int_0^4 \frac{x^2}{2}\Big|_0^{\sqrt{4-y}} \cdot \frac{e^{2y}}{4-y}\, dy$$

$$= \int_0^4 \frac{\cancel{4-y}}{2} \cdot \frac{e^{2y}}{\cancel{4-y}}\, dy = \int_0^4 \frac{1}{2} e^{2y}\, dy = \frac{1}{4} e^{2y}\Big|_0^4$$

$$= \frac{1}{4}\left(e^8 - 1\right)$$

10. (b) Calcule a área da região do plano limitada pelas curvas
$$y = e^x \,,\quad y = \operatorname{sen} x \,,\quad x = \pi \;\text{ e }\; x = -\pi \,.$$

Solução :




$$\text{área} = \int_{-\pi}^{\pi}\int_{\operatorname{sen} x}^{e^x} dy\,dx = \int_{-\pi}^{\pi}(e^x - \operatorname{sen} x)\,dx$$

$$= e^x\Big|_{-\pi}^{\pi} + \cos x\Big|_{-\pi}^{\pi} = e^{\pi} - e^{-\pi} + 0 = e^{\pi} - \frac{1}{e^{\pi}}$$

$$= \frac{e^{2\pi} - 1}{e^{\pi}}$$