LISTA 3

① $\int_C (xy + y + z)\,ds$ ao longo da curva
$\vec{r}(t) = 2t\,\vec{i} + t\,\vec{j} + (2-2t)\,\vec{k}$
$0 \leq t \leq 1$

solução :

$\vec{r}\,'(t) = 2\vec{i} + \vec{j} - 2\vec{k}$

$\|\vec{r}\,'(t)\| = \sqrt{4+1+4} = \sqrt{9} = 3$

$ds = 3\,dt$

$$\int_C (xy+y+z)\,ds = \int_0^1 \left(2t^2 + t + (2-2t)\right) 3\,dt$$
$$= 3\int_0^1 (2t^2 - t + 2)\,dt$$
$$= 3\left(\frac{2t^3}{3} - \frac{t^2}{2} + 2t\right)\Big|_0^1$$
$$= 3\left(\frac{2}{3} - \frac{1}{2} + 2\right) = 3\left(\frac{4-3+12}{6}\right)$$
$$= 13/2$$

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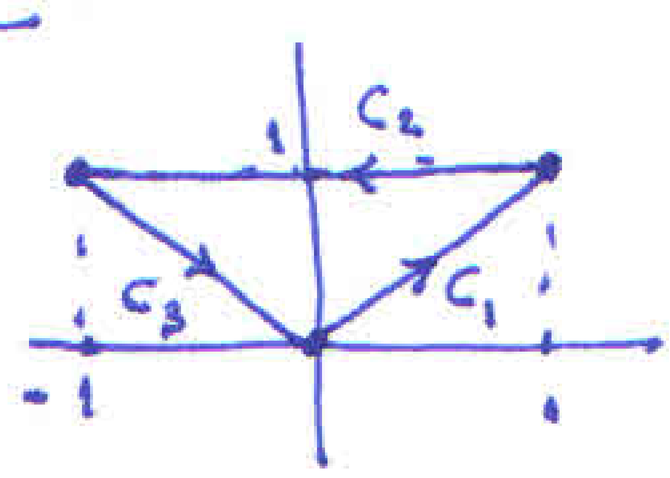
② $\int_C (x+y)\,ds$, C é o triângulo de vértices $(0,0)$, $(1,1)$ e $(-1,1)$

solução:



$C_1: \quad \vec{r}(t) = (t, t), \quad t \in [0,1], \quad \vec{r}'(t) = (1,1), \quad \|\vec{r}'\| = \sqrt{2}$

$C_2: \quad \vec{r}(t) = (1-2t, 1), \quad t \in [0,1], \quad \vec{r}'(t) = (-2,0), \quad \|\vec{r}'\| = \sqrt{4} = 2$

$C_3: \quad \vec{r}(t) = (t-1, 1-t), \quad t \in [0,1], \quad \vec{r}'(t) = (1,-1), \quad \|\vec{r}'\| = \sqrt{2}.$

$$\int_C (x+y)\,ds = \int_{C_1} (x+y)\,ds + \int_{C_2} (x+y)\,ds + \int_{C_3} (x+y)\,ds$$

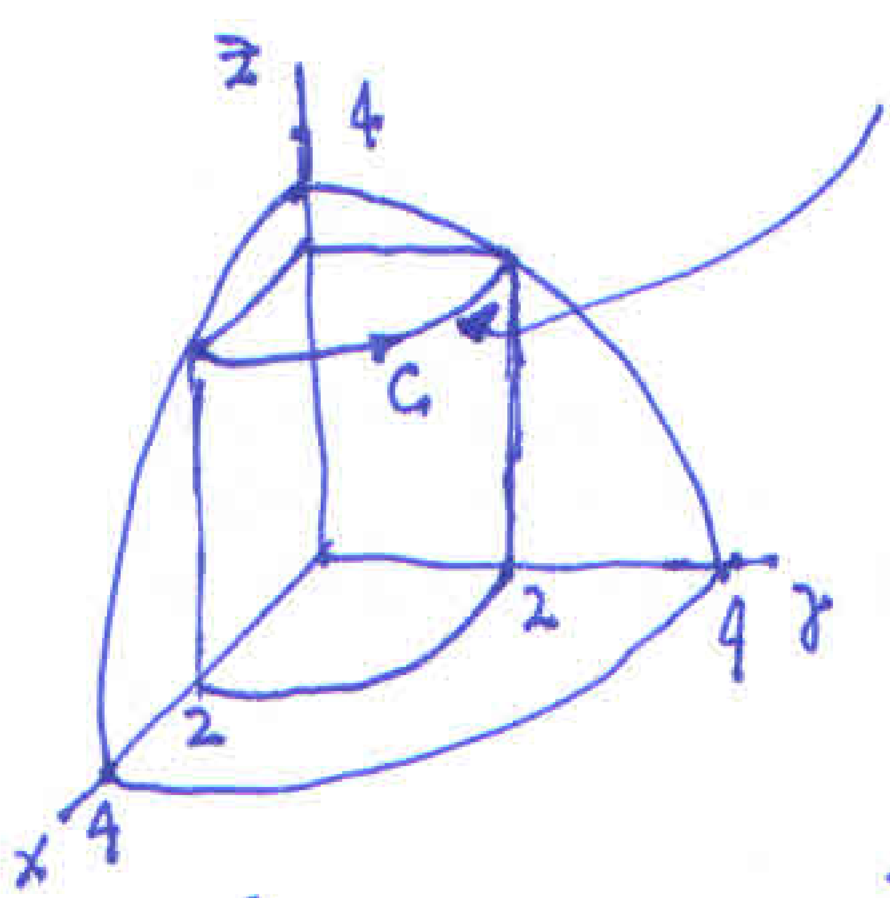
$$= \int_0^1 (t+t)\sqrt{2}\,dt + \int_0^1 (\cancel{1}-2t+\cancel{1})2\,dt + \int_0^1 (t-1+1-t)\sqrt{2}\,dt$$

$$= \int_0^1 2\sqrt{2}\,t\,dt + \int_0^1 2(2-2t)\,dt + \cancel{\int_0^1 0\,dt} \to 0$$

$$= \cancel{2}\sqrt{2} \cdot \frac{t^2}{\cancel{2}}\Big|_0^1 + \left(4t - \frac{4t^2}{2}\right)\Big|_0^1$$

$$= \sqrt{2} + 4 - 2 = \sqrt{2} + 2$$

3) $\int_C \sqrt{3}\, xyz\, ds$. C é a curva interseção da esfera $x^2+y^2+z^2=16$ com o cilindro $x^2+y^2=4$ (no 1º octante)



$$\begin{cases} z = 2\sqrt{3} \\ x^2+y^2 = 4 \end{cases}$$

$\vec{r}(t) = (2\cos t, 2\,\text{sen}\,t, 2\sqrt{3})$

$0 \le t \le \pi/2, \quad \vec{r}'(t) = (-2\,\text{sen}\,t, 2\cos t, 0)$

$\|\vec{r}'(t)\| = \sqrt{4} = 2$

$$\int_C \sqrt{3}\, xyz\, ds = \int_0^{\pi/2} \sqrt{3}\; 4\,\text{sen}\,t\cos t \cdot 2\sqrt{3} \cdot 2\,dt$$

$$\int_C \sqrt{3}\, xyz\, ds = \int_0^{\pi/2} 48 \operatorname{sen} t \cos t \, dt$$

$$= \frac{48 \operatorname{sen}^2 t}{2} \Big]_0^{\pi/2} = \frac{48}{2} = 24$$

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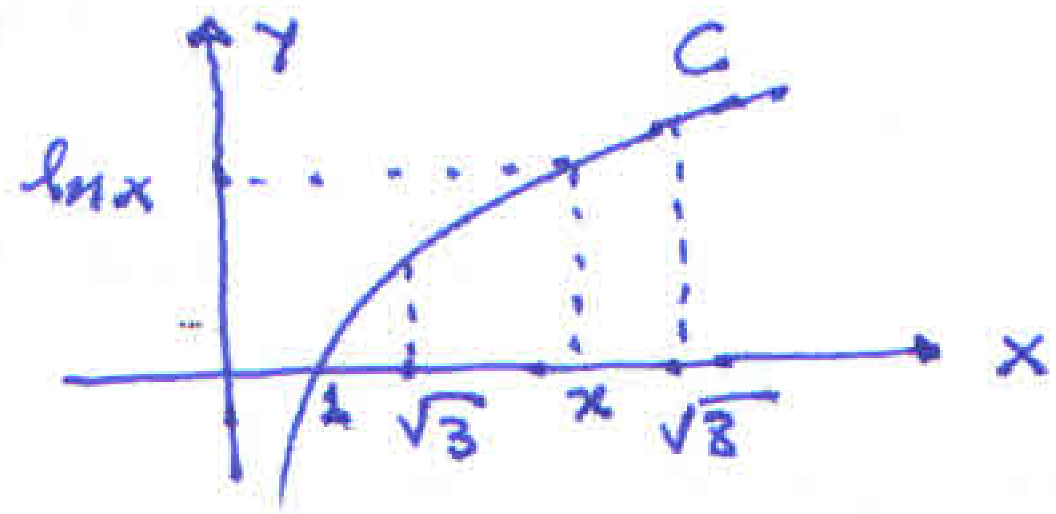
$$y = \ln x \ , \quad \sqrt{3} \le x \le \sqrt{8}$$

$$\delta(x,y) = x^2$$



$$C: \ \vec{r}(t) = (t, \ln t) \ , \quad t \in [\sqrt{3}, \sqrt{8}]$$

$$\vec{r}\,'(t) = \left(1, \frac{1}{t}\right) \ , \quad \|\vec{r}\,'(t)\| = \sqrt{1 + \frac{1}{t^2}} = \frac{1}{t}\sqrt{1+t^2}$$

$$\text{Massa} = M = \int_C \delta(x,y)\, ds$$

$$= \int_C x^2\, ds = \int_{\sqrt{3}}^{\sqrt{8}} t^2 \cdot \frac{1}{t} \sqrt{1+t^2}\, dt$$

$$= \int_{\sqrt{3}}^{\sqrt{8}} t \sqrt{1+t^2}\, dt \ , \qquad \begin{array}{l} u = 1+t^2 \\ du = 2t\,dt \\ t = \sqrt{3} \Rightarrow u = 4 \\ t = \sqrt{8} \Rightarrow u = 9 \end{array}$$

$$= \int_4^9 \frac{1}{2}\sqrt{u}\, du = \frac{1}{3} u^{3/2} \Big|_4^9$$

$$= 9 - 8/3 = \frac{19}{3} \ \text{unidades de massa}$$

(5) Massa do arame com a forma da hélice parametrizada por

$$\alpha : [0, \pi/2] \longrightarrow \mathbb{R}^3, \quad \alpha(t) = (3\cos t, 3\,\text{sen}\, t, 4t)$$

com densidade $\quad \delta(x,y,z) = \dfrac{kx}{1+y^2}$

$$\text{Massa} = M = \int_\alpha \delta(x,y,z)\, ds \,.$$

$$ds = \|\alpha'(t)\|\, dt$$

$$\alpha'(t) = (-3\,\text{sen}\, t, 3\cos t, 4), \quad \|\alpha'(t)\| = \sqrt{9+16} = \sqrt{25} = 5$$

$$\therefore \; \underline{ds = 5\,dt}$$

$$M = \int_0^{\pi/2} k \, \frac{3\cos t}{1+9\,\text{sen}^2 t} \cdot 5\,dt = 5k \int_0^{\pi/2} \frac{3\cos t}{1+9\,\text{sen}^2 t}\, dt$$

$$\begin{cases} u = 3\,\text{sen}\, t, \quad du = 3\cos t\, dt \\ t = 0 \Rightarrow u = 0 \\ t = \pi/2 \Rightarrow u = 3 \end{cases}$$

$$M = 5k \int_0^3 \frac{du}{1+u^2} = 5k \, \text{arctg}\, u \, \Big|_0^3$$

$$= 5k\, \text{arctg}(3)$$

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(a) $\vec{F}(x,y,z) = (\operatorname{sen} xy, \cos xy, z) = (P, Q, R)$

$$\operatorname{div}(\vec{F}) = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$\frac{\partial P}{\partial x} = y \cos xy, \quad \frac{\partial Q}{\partial y} = -x \operatorname{sen} xy, \quad \frac{\partial R}{\partial z} = 1$$

$$\therefore \underline{\operatorname{div}(\vec{F}) = y \cos xy - x \operatorname{sen} xy + 1}$$

$$\operatorname{rot}(\vec{F}) = \det \begin{pmatrix} \vec{i} & \vec{j} & \vec{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{pmatrix}$$

$$= \left(\frac{\partial R}{\partial y} - \frac{\partial Q}{\partial z}\right)\vec{i} + \left(\frac{\partial P}{\partial z} - \frac{\partial R}{\partial x}\right)\vec{j} + \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right)\vec{k}$$

$$= (0-0)\vec{i} + (0-0)\vec{j} + (-y \operatorname{sen} xy - x \cos xy)\vec{k}$$

$$= \underline{-(x \cos xy + y \operatorname{sen} xy)\vec{k}}$$