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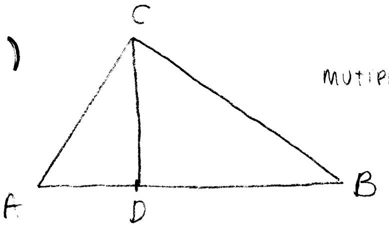
1-)




$\vec{DB} = 5\vec{AD}$

MULTIPLICA POR 5 × $\vec{AD} = -\vec{CA} + \vec{CD}$

$5\vec{AD} = -5\vec{CA} + 5\vec{CD}$

$\vec{DB} = -\vec{CD} + \vec{CB}$

COMO $DB = 5AD$

$5\vec{AD} = -\vec{CD} + \vec{CB}$

$-5\vec{CA} + 5\vec{CD} = -\vec{CD} + \vec{CB}$

$6\vec{CD} = \vec{CB} + 5\vec{CA}$

$\boxed{\vec{CD} = \frac{\vec{CB}}{6} + \frac{5\vec{CA}}{6}}$

1,0

2-)

$(\vec{a}, \vec{b}) = 60°$  $\|\vec{a}\| = 10$

$(\vec{a}, \vec{c}) = 60°$  $\|\vec{b}\| = 8$

$(\vec{b}, \vec{c}) = 90°$  $\|\vec{c}\| = 5$

$(2\vec{a} + \vec{b})(\vec{a} - 2\vec{c}) = 2.\vec{a}.\vec{a} - 4.\vec{a}.\vec{c} + \vec{b}.\vec{a} - \underbrace{2\vec{b}.\vec{c}}_{\text{PERPENDICULARES}}$ 0

$2\|\vec{a}\|^2 - 4\|\vec{a}\|\|\vec{c}\|.\cos(\vec{a}, \vec{c}) + \|\vec{b}\|\|\vec{a}\|.\cos(\vec{b}, \vec{a})$   cos 90° = 0

$200 - 4.10.5.\frac{1}{2} + 8.10.\frac{1}{2}$

$200 - 100 + 40 = 100 + 40 = \boxed{140}$

2,0

3-) $\vec{a} = \vec{i} + \vec{j} + \vec{k}$

$\vec{b} = 2\vec{i} + \vec{j} - 3\vec{k}$

$\vec{c} = -4\vec{i} + 5\vec{j} - \vec{k}$

a) Se é base em $R^3$, o determinante de seus coeficientes tem que ser ≠ 0

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & -3 \\ -4 & 5 & -1 \end{vmatrix} \begin{matrix} 1 & 1 \\ 2 & 1 \\ -4 & 5 \end{matrix} = -1 + 12 + 10 - (-4 - 15 - 2)$$

$-1 + 22 + 4 + 15 + 2 = -1 + 43 = 42$

$Det \neq 0$, pois $= 42$, logo é base em $R^3$.

$\vec{a}.\vec{b} = \|\vec{a}\|\|\vec{b}\|.\cos(\vec{a}, \vec{b})$ $0 \to 90°$

Se é ortogonal, temos que

$\vec{a}.\vec{b} = 0$
$\vec{b}.\vec{c} = 0$
$\vec{a}.\vec{c} = 0$
pois cos 90° = 0 (perpendiculares)

• $\vec{a}.\vec{b} = a_1.b_1 + a_2.b_2 + a_3.b_3$
$= 1.2 + 1.1 + 1.(-3)$
$2 + 1 - 3 = 0$

$\boxed{\vec{a}.\vec{b} = 0}$

• $\vec{b}.\vec{c} = b_1.c_1 + b_2.c_2 + b_3.c_3$
$2.(-4) + 1.5 + (-3).(-1)$
$-8 + 5 + 3 = 0$

$\boxed{\vec{b}.\vec{c} = 0}$

• $\vec{a}.\vec{c} = a_1.c_1 + a_2.c_2 + a_3.c_3$
$= 1.(-4) + 1.5 + 1.(-1)$
$-4 + 5 - 1 = 0$

$\boxed{\vec{a}.\vec{c} = 0}$

→ Provando assim que eles formam uma base ortogonal.

1,0

b) $\vec{v} = 4\vec{i} + 10\vec{j} - 8\vec{k}$

$\vec{v} = x\vec{a} + y\vec{b} + z\vec{c}$

$\vec{v} = x(\vec{i} + \vec{j} + \vec{k}) + y(2\vec{i} + \vec{j} - 3\vec{k}) + z(-4\vec{i} + 5\vec{j} - \vec{k})$

$\vec{v} = (x + 2y - 4z)\vec{i} + (x + y + 5z)\vec{j} + (x - 3y - z)\vec{k} = 4\vec{i} + 10\vec{j} - 8\vec{k}$

$$\begin{cases} x + 2y - 4z = 4 \\ x + y + 5z = 10 \\ x - 3y - z = -8 \end{cases}$$

$$\left|\begin{matrix} 1 & 2 & -4 & 4 \\ 1 & 1 & 5 & 10 \\ 1 & -3 & -1 & -8 \end{matrix}\right| \begin{matrix} \\ L_2 \to L_2 - L_1 \\ L_3 \to L_3 - L_1 \end{matrix} = \left|\begin{matrix} 1 & 2 & -4 & 4 \\ 0 & -1 & 9 & 6 \\ 0 & -5 & 3 & -12 \end{matrix}\right| \to L_3 = L_3 + [(-5)L_2] =$$

$$\left|\begin{matrix} 1 & 2 & -4 & 4 \\ 0 & -1 & 9 & 6 \end{matrix}\right|$$

• $-42z = -42$

$\boxed{z = 1}$

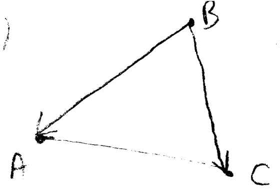
$x = 6 - 4 = 2$
$y = 2$

1,0

Logo

$\boxed{\vec{v} = 2\vec{a} + 3\vec{b} + \vec{c}}$

4-) a)




$A = (1, 2, -1)$

$B = (2, 4, 1)$

$C = (4, 2, 2)$

a) $\vec{BA} = -\vec{i} - 2\vec{j} - 2\vec{k}$

$\vec{BC} = 2\vec{i} - 2\vec{j} + \vec{k}$

Para ser retângulo, tem que ter um ângulo de 90°

$\cos(\vec{BA}, \vec{BC}) = \dfrac{\vec{BA} \cdot \vec{BC}}{\|\vec{BA}\| \|\vec{BC}\|} = \dfrac{(-1) \cdot 2 + (-2)(-2) + (-2) \cdot 1}{\|\vec{BA}\| \|\vec{BC}\|}$

$\cos(\vec{BA}, \vec{BC}) = \dfrac{0}{\|\vec{BA}\| \|\vec{BC}\|} = 0 \quad \cos 90° = 0$

Logo o ângulo é de 90° por isso retângulo.

1,0

b) $A_\Delta = \dfrac{\|\vec{BC} \times \vec{BA}\|}{2} =$

$\vec{BC} \times \vec{BA} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & -2 & -2 \\ 2 & -2 & 1 \end{vmatrix}$

1,0

$\vec{BC} \times \vec{BA} = -6\vec{i} - (3\vec{j}) + 6\vec{k}$

$\|\vec{BC} \times \vec{BA}\| = \sqrt{(-6)^2 + (-3)^2 + (6)^2} = \sqrt{81} = 9$

$A_{\Delta ABC} = \dfrac{9}{2} = 4{,}5$ u.a.

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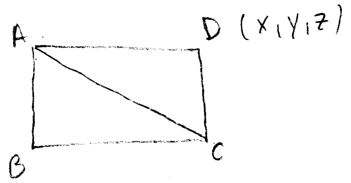
c)




Se é um retângulo logo $\vec{AD}$ é igual a $\vec{BC}$

$\vec{AD} = \vec{BC}$

$(x-1)\vec{i} + (y-2)\vec{j} + (z+1)\vec{k} = 2\vec{i} - 2\vec{j} + \vec{k}$

$(x-1) = 2 \quad x = 3$

$y - 2 = -2 \quad y = 0$

$z + 1 = 1 \quad z = 0$

$D = (3, 0, 0)$

1,0

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5-) a) $\vec{c} = \vec{a} \times \vec{b}$ para ser perpendicular a $\vec{a}$ e $\vec{b}$.

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -3 \\ 1 & -1 & 2 \end{vmatrix} = -\vec{i} - (7\vec{j}) - 3\vec{k}$

$\vec{c} = -\vec{i} - 7\vec{j} - 3\vec{k}$

1,0

b) Para $\vec{d}$ ser perpendicular a $\vec{a}$ e $\vec{c}$ tem-se que $\vec{d} = \vec{a} \times \vec{c}$

$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & -3 \\ -1 & -7 & -3 \end{vmatrix} = -24\vec{i} - (-9\vec{j}) - 13\vec{k}$

$\vec{d} = -24\vec{i} + 9\vec{j} - 13\vec{k}$

1,0

c) $\vec{d} = x\vec{a} + y\vec{b}$

$\vec{d} = x(2\vec{i} + \vec{j} - 3\vec{k}) + y(\vec{i} - \vec{j} + 2\vec{k})$

$(2x + y)\vec{i} + (x - y)\vec{j} + (-3x + 2y)\vec{k}$

$\begin{cases} 2x + y = -24 \\ x - y = 9 \\ -3x + 2y = -13 \end{cases}$

$x = 9 + y$

$2(9 + y) + y = -24$

$18 + 2y + y = -24$

$3y = -24 - 18$

$y = -8 - 6$

$x + 14 = 9$

$x = -5$

1,0

Logo o vetor

$\vec{d} = -5\vec{a} - 14\vec{b}$