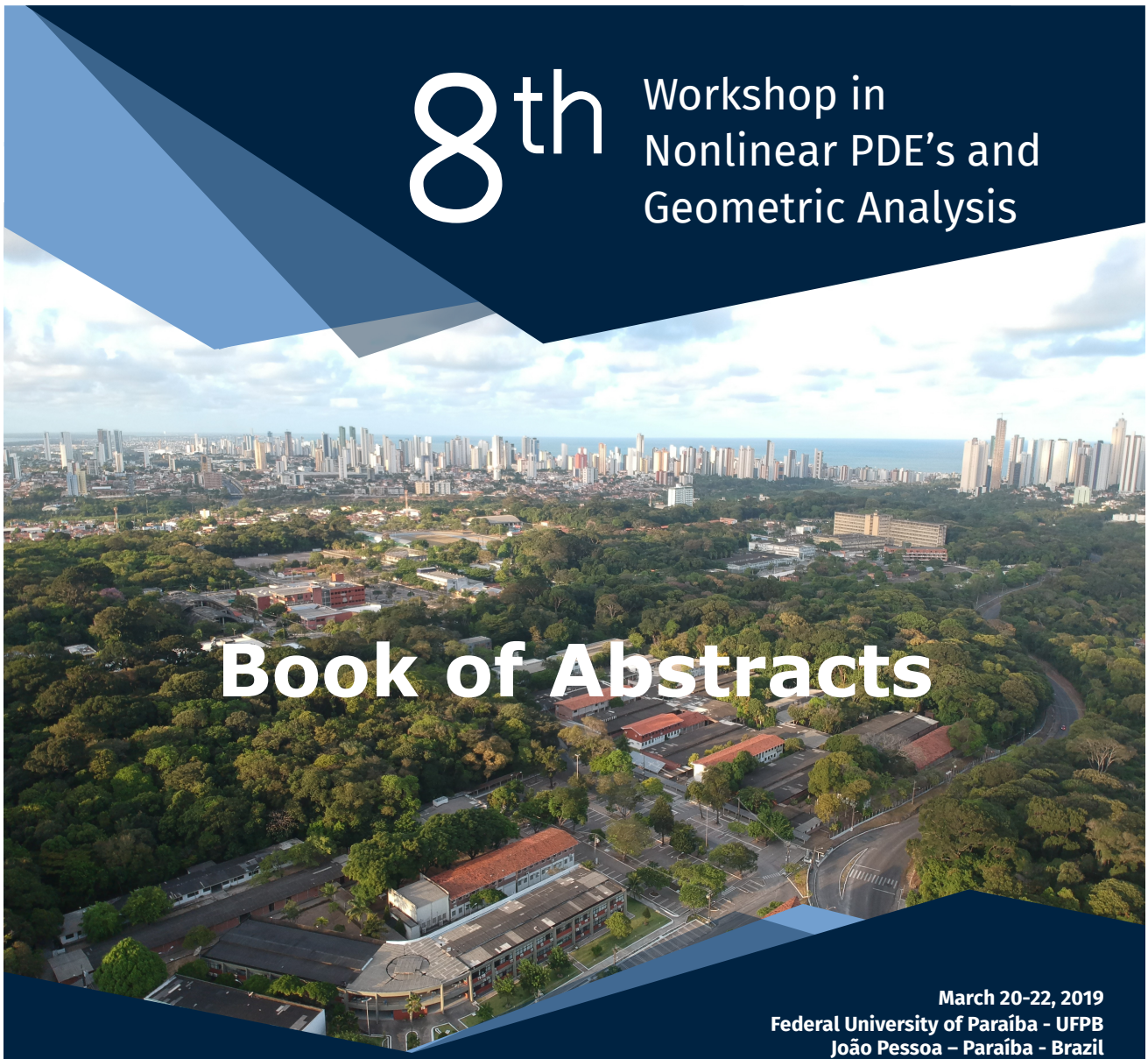
8th Workshop in Nonlinear PDE's and Geometric Analysis



Book of Abstracts

March 20-22, 2019
Federal University of Paraíba - UFPB
João Pessoa – Paraíba - Brazil

Local Organizing Committee

Bruno Ribeiro
Damião J. Araújo
Elisandra Gloss
Everaldo Medeiros
Fagner Araruna
Felipe W. Chaves-Silva
Flank Bezerra
Gaetano Siciliano
Manassés X. de Souza
Maurício C. Santos
Uberlandio Severo

Scientific Committee

Bernhard Ruf – Univ. of Milan, Italy
Djairo de Figueiredo – Unicamp, Brazil
Eduardo Teixeira – Univ. of Central Florida, USA
Enrique Fernández-Cara – Univ. of Sevilla, Spain
João Marcos do Ó – UnB, Brazil

www.mat.ufpb.br/wenlu

Support

Universidade Federal da Paraíba

VIII WORKSHOP IN NONLINEAR PDE'S AND GEOMETRIC ANALYSIS

Universidade Federal da Paraíba
Centro de Ciências Exatas e da Natureza
Departamento de Matemática
58051-900, João Pessoa – PB

Rector: Margareth de Fátima Formiga Melo Diniz
Director of Center: José Roberto Soares do Nascimento
Head of Department: Joedson Silva dos Santos

Scientific Committee

Djairo Guedes de Figueiredo (UNICAMP, Brazil)
João Marcos Bezerra do Ó (UnB, Brazil)
Enrique Fernández-Cara (Universidad de Sevilla, Spain)
Eduardo Vasconcelos Oliveira Teixeira (University of Central Florida, USA)
Bernhard Ruf (Universit`a di Milano, Italy)

Organizing Committee

Bruno Henrique Carvalho Ribeiro (UFPB)
Damião Junio Gonçalves Araújo (UFPB)
Elisandra de Fátima Gloss de Moraes (UFPB)
Everaldo Souto de Medeiros (UFPB)
Gaetano Siciliano (USP)
Fágner Dias Araruna (UFPB)
Felipe W. Chaves-Silva (UFPB)
Flank David Morais Bezerra (UFPB)
Manassés Xavier de Souza (UFPB)
Maurício Cardoso Santos (UFPB)
Uberlandio Batista Severo (UFPB)

ABOUT THE WORKSHOP

The Workshop in Nonlinear Partial Differential Equations and Geometric Analysis is by now a familiar conference among researchers in the areas. Originally called Workshop em Equações Não Lineares da UFPB (WENLU), the event now includes participants from several countries and therefore a redesign in its structure (and even the name) was required. This is the 8th edition and it will be held in the new audithorium of the Mathematics Department of UFPB.

The main objective of this event is to bring together researchers to disseminate their work in progress, allowing an exchange of ideas among experts and exposing students and recent PhDs to research themes that have been developed in several institutions from Brazil and abroad.

The organizing committee of the *VIII Workshop in Nonlinear PDE's and Geometric Analysis*wishes to express their gratitude to the institutions that supported and made possible the realization of this event: UFPB, CAPES, CNPq, FAPESP, INCTMat and PMJP. Thanks also to all the attendees, as well as to the employees for their enthusiasm and effort, which were important to the development of all the event activities.

The organizing committee

Summary

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

On a Hamiltonian system with critical growth

Angelo Guimarães *

In this work we study the Hamiltonian system

$$\begin{cases} -\Delta u = \lambda|v|^{r-1}v + |v|^{p-1}v \text{ in } \Omega, \\ -\Delta v = \mu|u|^{s-1}u + |u|^{q-1}u \text{ in } \Omega, \\ u, v = 0 \text{ on } \partial\Omega, \end{cases} \tag{S}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, $N \geq 3$, $\lambda > 0$ and $\mu > 0$. We assume that (p, q) lies on the critical hyperbola, that is

$$\frac{1}{p+1} + \frac{1}{q+1} = \frac{N-2}{N}$$

and (r, s) satisfies

$$0 < r < p, \quad 0 < s < q, \quad rs \geq 1.$$

In this paper we classify regions on the critical hyperbola as critical and non-critical. We extend to the framework of these systems the classification of critical and non-critical dimensions introduced in the pioneering work of Brezis and Nirenberg on second order elliptic equations.

Joint work with Ederson M. dos Santos (ICMC-USP), Jéssyca L. F. Melo (DM-UFV) and Marcelo F. Furtado (DM-UnB).

*Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, email: g.angelo@usp.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

On the Riemann problem for a 2×2 system of conservation laws

Aparecido de Souza *

In this talk we discuss the solution of the Riemann problem for a 2×2 system of non-strictly hyperbolic conservation laws that model a three-phase flow in porous media. This system fails to be strictly hyperbolic at an umbilic point, i.e. an isolated point interior to state space at which the characteristic speeds coincide. Using the viscous profile admissibility for shock waves we combine numerical and analytical techniques to provide scientific evidence for the existence, uniqueness and L^1_{Loc} continuity of solutions under change of data and parameters.

The solution consists of at most three groups of self-similar waves separated by two constant states. Depending on the initial data, and on the fluid viscosities, two structures arise for the solution. In the first structure, the solution comprises at most two classical (in the sense of Lax) wave groups and in the second it comprises two classical and one nonclassical wave groups.

Joint work with Patrício Andrade (IFPB), Arthur Azevedo (UnB), Luciano Barros (UFCG), Frederico Furtado (University of Wyoming), M. Joseane Guedes (UFERSA), Luis Lozano (UFJF) and Dan Marchesin (IMPA).

*Department of Scientific Computing, UFPB, email: aparecidosouza@ci.ufpb.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

A fractional reaction-diffusion equation

Arlúcio Viana *

We shall discuss the existence and nonexistence of global positive solutions for the semilinear fractional diffusion equation

$$u_t(t,x) = \int_0^t dg_\alpha(s)\Delta u(t-s,x) + |u(t,x)|^{\rho-1}u(t,x), \text{ in } (0,\infty)\times\mathbb{R}^N; \tag{0.1}$$

$$u(x,0) = u_0(x), \text{ in } \mathbb{R}^N, \tag{0.2}$$

where $\rho > 1$. Indeed, a combination of [1] and [2] provides the following result:

1. Let $u_0 \in L^1(\mathbb{R}^N, e^{-\frac{|x|^2}{4}})$ be nonnegative, $u \neq 0$ and $1 < \rho \leq 1 + \frac{2}{\alpha N}$, then there exists no bounded positive global solution of (0.1)-(0.2).
2. Let $\rho > 1 + \frac{2}{\alpha N}$ and $u_0 \in L^{\frac{\alpha N}{2}(\rho-1)}(\mathbb{R}^N)$ with $\|u_0\|_{L^{\frac{\alpha N}{2}(\rho-1)}(\mathbb{R}^N)}$ sufficiently small, then there exists a positive global solution of (0.1)-(0.2).

Eventually, we highlight some asymptotic properties of the existing global solution as well as we exhibit a local well-posedness result in L^p.

References

[1] B. de Andrade and A. Viana, On a fractional reaction-diffusion equation, Z. Angew. Math. Phys. **68** (2017), no. 3, Art. 59, 11 pp.
[2] A. Viana, A local theory for a fractional reaction-diffusion equation, Comm. Cont. Math., to appear, doi: 10.1142/S0219199718500335.

*Department of Mathematics, Federal University of Sergipe, email: arlucioviana@gmail.com

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Decay rate for waves with indefinite memory

Bianca M.R. Calsavara *

In the context of partial differential equations systems where the dissipative effects are given by memory terms and they change sign there are few studies about the energy decay rate. One of the earliest studies in this direction is due to Muñoz-Rivera and Naso. They considered the following functional equation with memory term

$$u_{tt} + Au - \int_0^t g(t-s)Au(s)ds = 0,$$

where the memory kernel g can change sign. They proved, in [1], the exponential decay of the solutions if $0 < g(0) < \lambda_1$, where λ_1 is the smaller eigenvalue of the self-adjoint, positive definite operator A in a Hilbert space. In this work, the memory dissipation is distributed on whole domain. Its a open study when this dissipation is distributed only in a part of its domain.

In this work we deal with the following wave equation with localized dissipation given by a memory term

$$u_{tt} - u_{xx} + \partial_x \left\{ a(x) \int_0^t g(t-s)u_x(x,s)ds \right\} = 0.$$

Here we consider that the dissipation is indefinite due to sign changes of the coefficient a or by sign changes of the memory kernel g. With this condition, we prove the exponential decay of solutions when the average of coefficient a is positive and the memory kernel g is small.

Joint work with Higidio P. Oquendo, *UFPR, Brazil.* **E-mail:** *higidio@ufpr.br.*

*IMECC, UNICAMP, Brazil. **E-mail:***bianca@ime.unicamp.br*

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Carleman estimates for some two-dimensional degenerate parabolic PDEs and applications

B. S. V. Araújo*

In this work, we will stablish null controllability results for the following problem in two spatial dimensions:

$$\begin{cases} u_t - \operatorname{div}(A\nabla u) + bu = g1_\omega & \text{in} \quad Q, \\ B.C. & \text{on} \quad \Sigma, \\ u(\cdot,0) = u_0 & \text{in} \quad \Omega, \end{cases} \tag{0.1}$$

where $\Omega := (0,1)\times(0,1)$, $\Gamma := \partial\Omega$, $T > 0$, $Q := \Omega \times (0,T)$, $\Sigma := \Gamma \times (0,T)$, $\omega \subset \Omega$ is a non-empty open set and 1_ω is the corresponding characteristic function, $b \in L^\infty(Q)$, $g \in L^2(Q)$, $u_0 \in L^2(\Omega)$, the matrix-valued function $A : \overline{\Omega} \mapsto M_{2\times 2}(\mathbb{R})$ is given by

$$A(x) = diag(x_1^{\alpha_1}, x_2^{\alpha_2}),$$

the boundary conditions are

$$B.C. := \begin{cases} u = 0 \ \text{ on } \ \Sigma & \text{if} \quad \alpha_1,\ \alpha_2 \in [0,1), \\ u = 0 \ \text{ on } \ \Sigma_{3,4} \ \text{ and } \ (A\nabla u)\nu = 0 \ \text{ on } \ \Sigma_{1,2} & \text{if} \quad \alpha_1,\ \alpha_2 \in [1,2], \\ u = 0 \ \text{ on } \ \Sigma_{1,3,4} \ \text{ and } \ (A\nabla u)\nu = 0 \ \text{ on } \ \Sigma_2 & \text{if} \quad \alpha_1 \in [0,1) \ \text{ and } \ \alpha_2 \in [1,2], \\ u = 0 \ \text{ on } \ \Sigma_{2,3,4} \ \text{ and } \ (A\nabla u)\nu = 0 \ \text{ on } \ \Sigma_1 & \text{if} \quad \alpha_1 \in [1,2] \ \text{ and } \ \alpha_2 \in [0,1), \end{cases}$$

with $\alpha = (\alpha_1, \alpha_2) \in [0,2]\times[0,2]$, $\Sigma_{i,j,l} := (\Gamma_i \cup \Gamma_j \cup \Gamma_l)\times(0,T)$, $\nu = \nu(x)$ being the outward unit normal to Ω at the point $x \in \Gamma$ and, finally,

$$\Gamma_1 := \{0\}\times[0,1],\ \Gamma_2 := [0,1]\times\{0\},\ \Gamma_3 := \{1\}\times[0,1],\ \Gamma_4 := [0,1]\times\{1\}.$$

The system (0.1) appear in financial problems of option pricing. Hance, controllability results for this system is a interesting question. This results can be obtained by the HUM method, which can be applied by using the Carleman estimates that we will prove in this work.

The main difficult of obtain Carleman estimates to (0.1) is that the differential operator degenerates only in a part of the boundary which is connected to the rest of border. The Carleman estimates present in literature consider differential oparators which degenerate on the whole border.

Joint work with F. D. Araruna (UFPB) and E. Fernández-Cara (Universidad de Sevilla).

*Dpto. de Matemática, Universidade Federal de Campina Grande, 58429-000, Campina Grande - PB, Brazil (bsergio@mat.ufcg.edu.br)

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Large time decay for solutions of the magneto-micropolar system

Cilon F. Perusato *

This work examines the large time decay of solutions to the viscous, incompressible magneto-micropolar equations in $\mathbb{R}^n$ $(n = 2, 3)$ under very general assumptions on the initial data, thus extending previous results obtained by a number of authors (T. Kato, M. Wiegner, M. Schonbek, P. Zingano and others) for the classic case of the Navier-Stokes equations. We use a new approach that quickly leads to the results and can be easily adapted to other important systems (including advection-diffusion equations, Burgers' equation and so on), so that it should be of interest to a large audience, particularly people working in partial differential equations and fluid dynamics. The main result is that, for arbitrary initial data $(\boldsymbol{u}_0, \boldsymbol{w}_0, \boldsymbol{b}_0) \in L^2(\mathbb{R}^n)$, with $\boldsymbol{u}_0$ and $\boldsymbol{b}_0$ divergence-free, solutions decay in $\dot{H}^s$ (for each $s \geq 0$ real), with

$$\lim_{t\to\infty} t^{s/2} \|(\boldsymbol{u}, \boldsymbol{w}, \boldsymbol{b})(\cdot, t)\|_{\dot{H}^s(\mathbb{R}^n)} = 0.$$

as $t \to \infty$. Moreover, in the presence of vortex viscosity, the micro-rotational field $\boldsymbol{w}(\cdot, t)$ decays faster, with

$$\lim_{t\to\infty} t^{\frac{s+1}{2}} \|\boldsymbol{w}(\cdot, t)\|_{\dot{H}^s(\mathbb{R}^n)} = 0. \tag{0.1}$$

as $t \to \infty$. As a consequence, for $s = m$ integer we get a decay result in $L^q(\mathbb{R}^n)$ $(1 \leq q \leq \infty$ and $n = 2, 3.)$ for each derivative of order m. Several related estimates are also obtained, including a new inequality for solutions of the magneto-micropolar system in $\mathbb{R}^n$ $(n = 2, 3)$.

*Department of Mathematics, Universidade Federal de Pernambuco, email: cilonperusato@gmail.com

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Bounded perturbations of some null-controllable systems

Diego A. Souza*

In this talk we present controllability results of general bounded perturbed exactly controlled linear systems with admissible control operators. Firstly, we show that approximate and null controllability are equivalent properties for such systems. Then, and more importantly, we provide for the perturbed system a complete characterization of the set of reachable states in terms of the Fattorini-Hautus test. The results rely on the perturbation theory of Kato.

Joint work with Guillaume Olive (Jagiellonian University).

*Department of Mathematics, Federal University of Pernambuco, email: diego.souza@dmat.ufpe.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

An optimization problem for the fractional Laplacian with a volume constraint and a lower temperature bound

Diego Marcon Farias *

We consider a free boundary optimization problem for the fractional Laplacian with a volume constraint and a lower temperature bound. We prove the existence and the optimal regularity of solutions. Moreover, two natural free boundaries arise in our problem. So, we provide not only geometric properties of solutions, but also of the corresponding exterior and interior free boundaries.

This is a joint work with V. Nersesyan (Universit Paris-Saclay) and R. Teymurazyan (University of Coimbra).

*Departamento de Matemática, Universidade Federal do Rio Grande do Sul.

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Existence and non-existence results of dead cores for fully nonlinear elliptic problems

Disson dos Prazeres *

In this talk we going to show existence/non-existence results of positive solutions for second order equations as follows

$$(\mathrm{P}_{a,q}) \qquad \begin{cases} |Du|^{\gamma} F(x, D^2u) + a(x)u^q(x) = 0 & \text{in } \Omega, \\ u \geq 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

on a bounded and smooth domain $\Omega \subset \mathbb{R}^N$. In our approach F is a fully nonlinear elliptic operator (with certain structural conditions), q is a non-negative sub-linear absorption term in relation to homogeneity of operator, and $a : \Omega \to \mathbb{R}$ is a sign-changing weight. We also analyse some results concerning existence of non-negative solutions having dead cores.

Joint work with Humberto Ramos (UNC;USACH) and João Vitor da Silva (UBA).

*Department of Mathematics, Universidade Federal de Sergipe, email: disson@mat.ufs.br

WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

GROUND AND BOUND STATE SOLUTIONS FOR QUASILINEAR ELLIPTIC SYSTEMS INCLUDING SINGULAR NONLINEARITIES AND INDEFINITE POTENTIALS

Edcarlos Domingos da Silva*

It is established existence of bound and ground state solutions for quasilinear elliptic systems driven by (Φ_1, Φ_2)-Laplacian operator. The main feature here is to consider quasilinear elliptic systems involving both nonsingular nonlinearities combined with indefinite potentials and singular cases perturbed by superlinear and subcritical couple terms. This framework permit us to use arguments based on Ambrosetti-Rabinowitz condition and variational methods for dierentiable functionals. By exploring the Nehari method and doing a fine analysis on the fibering map associated, we get estimates that allow us unify the arguments to show multiplicity of semi- trivial solutions in both cases.

This is a jointly work with M. L. M Carvalho, C. A. Santos, C. Goulart.

*Universidade Federal de Goias, IME, Goiania-GO, Brazil E-mail address: edcarlos@ufg.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Some classical elliptic equations with nonlinearities with Hénon weight in the critical Trudinger-Moser growth range

Eudes Mendes Barboza *

Using variational methods we have investigated the existence of solutions for some Hénon type equations, which are characterized by the presence of the weight $|x|^\alpha$ in the nonlinearity with $\alpha > 0$. When we are working in the radial context, this characteristic modifies the critical growth of the nonlinearities in some senses. This fact allows us to study some well-known problems under new perspectives. For this purpose, we have considered two different classes of problems with critical nonlinearity which presents the weight of Hénon. Firstly, we have studied the class of problem with a Trudinger-Moser nonlinearity in critical range in $\mathbb{R}^2$. In the subcritical case, there was no difference if we have looked for weak solutions in $H_0^1(B_1)$ or in $H_{0,\text{rad}}^1(B_1)$. Nevertheless, in the critical case we have needed to adapt some hypotheses when we have changed the space where we were seeking the solutions. For the second problem, we have kept working with exponential nonlinearity in $\mathbb{R}^2$, but we were treating an Ambrosseti-Prodi problem for which we have searched two weak solutions. In the subcritical case, analogously to first problem, the radially symmetric solutions were obtained as the solutions in $H_0^1(B_1)$, what have not happened in the critical case. Thus, again some assumptions have had to depend on the context where we were searching for the solutions.

Joint work with João do Ó (UnB) and Bruno Ribeiro (UFPB).

References

[1] A. Ambrosetti, G. Prodi, *On the inversion of some differentiable mappings with singularities between Banach spaces*, Annali Mat. Pura Appl. **93** (1972) 231-247.

[2] Adimurthi, *Existence of positive Solutions of the semilinear Dirichlet Problem with critical growth for the n-Laplacian*, Ann. Sc. Norm. Sup. Pisa Cl. Sci. **17** (1990) 393-413.

[3] E. Barboza, J. do Ó, B. Ribeiro, *Hénon type equations with one-sided exponential growth*, Topological Methods in Nonlinear Analysis, DOI: 10.12775/TMNA.2017.010.

[4] M. Calanchi, B. Ruf, *Elliptic equations with one-sided critical growth*, Electronic Journal of Diff. Equations. **2002**(2002) 1-21.

[5] M. Calanchi, B. Ruf, Z. Zhang, *Elliptic equations in* $\mathbb{R}^2$ *with one-sided exponential growth*, Commun. Comtemp. Math. **6**(2004) 947-971.

[6] M. Calanchi, E. Terraneo, *Non-radial maximizer for functionals with exponential non-linearity in* $\mathbb{R}^2$, Adv. Nonlinear Stud. **5**(2005) 337-350.

[7] D. Figueiredo, O. Miyagaki, B. Ruf, *Elliptic equations* $\mathbb{R}^2$ *with nonlinearities in the critical growth range*, Calc. Var. and PDEs. **3** (1995) 139-153.

[8] D. Figueiredo, J. do Ó, B. Ruf, *On an inequality by N. Trudinger and J. Moser and related elliptic equations*, Comm. Pure Appl. and Math. **55** (2002) 135-152.

*Department of Mathematics, Federal Rural University of Pernambuco, email: eudesmendesbarboza@gmail.com

WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

On generalized derivative nonlinear Schrödinger equations

Felipe Linares*

In this talk we will present recent results concerning the initial value problem associated to generalized derivative nonlinear Schrödinger equations. We show local well-posedness for small initial data in a suitable weighted Sobolev spaces. We use an argument introduced by Cazenave and Naumkin to obtain our main results combined with the homogeneus and inhomogeneous smoothing efects of Kato type. If time permits we will show how these results can be extended for any data size in a suitable class.

Joint work with: Gustavo Ponce (UC Santa Barbara, USA), Gleison Santos (UFPI, Brazil).

*IMPA, Rio de Janeiro, Brazil, email: linares@impa.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Adams' type inequality and application to a quasilinear nonhomogeneous equation with singular and vanishing radial potentials in $\mathbb{R}^4$

Francisco Sibério B. Albuquerque *

In this talk, we establish some Adams' type inequality for weighted second order Sobolev spaces in four dimensions. The weights are radial and can have a singular or decaying behavior. This inequality is used to study some nonhomogeneous quasilinear elliptic equation.

Joint work with Sami Aouaoui (Institut Supérieur des Mathématiques Appliquées et de l'Informatique de Kairouan).

*Department of Mathematics, State University of Paraíba, email: fsiberio@cct.uepb.edu.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

On a non-isothermal flow for two-phase fluids

Gabriela Planas *

In this talk, we consider some models for a non-isothermal flow for two-phase incompressible fluids. This kind of model consists of modified Navier-Stokes equations coupled with a phase-field equation given by a convective Allen-Cahn equation, and energy transport equation for the temperature. We discuss questions related to the well-posedness of the problems in the two and three dimensional cases.

Joint work with Juliana H. Lopes (UFRJ).

*Departmento de Matemática, IMECC, Unicamp, email: gplanas@ime.unicamp.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Symmetry of positive solutions for fully nonlinear elliptic systems

Gabrielle Saller Nornberg *

In this talk we will discuss some recent symmetry results for positive solutions of fully nonlinear uniformly elliptic systems up to quadratic growth in the gradient. We develop a unified treatment of the classical moving planes method in the spirit of Gidas-Ni-Nirenberg, which permits us to work without Lipschitz assumptions on the zero order term dependence. We also present different applications of our results, including uniqueness of positive solutions for Lane-Emden systems in the subcritical case in a ball, and symmetry for a class of equations with natural growth in the gradient.

Joint work with Ederson Moreira dos Santos (ICMC-Usp).

*ICMC, Universidade de São Paulo email: gabrielle@icmc.usp.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Existence results for Kirchhoff type problems

Gaetano Siciliano *

We discuss, by means of change of variables and bifurcation methods, the existence of positive solutions for the following Kirchhoff type problem

$$\begin{cases} -\Delta(g(|\nabla u|_2^2)u + u^r) = u + u^p & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

in a bounded and smooth domain Ω in $\mathbb{R}^N$, where $p, r > 1$ and g is a suitable continuous function.

Joint work with João R. Santos Junior.

*Department of Mathematics, University of São Paulo, email: sicilian@ime.usp.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

STACKELBERG-NASH NULL CONTROLLABILITY OF PARABOLIC EQUATIONS IN UNBOUNDED DOMAINS

Gilcenio Rodrigues de Sousa Neto *

This work deal with a hierarchical control problem for a parabolic equation in a unbounded domain Ω following a Stackelberg-Nash strategy. We assume that there is a main (leader) control and two (followers) secondary control. The leader tries to drive the solution to a prescribed target and the followers intend to be a Nash equilibrium for given functionals. The result achieved so far expands the results of [1] for a class of unbounded domains. Namely, the unbounded domains whose control region $\mathcal{O}$ is such that $\Omega\backslash\mathcal{O}$ is bounded.

Joint work in progress with Vieira, F.B.(UFPI) and Jesus, I. P. (UFPI)

References

[1] Araruna FD, Fernández-Cara E, Guerrero S, Santos MC. New results on the Stackelberg-Nash exact control of linear parabolic equations. Systems & Control Letters. 2017; 104:78–85.

*CAA, UFPE, email: gilceniorodrigues@gmail.com

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Equações de Schrödinger Quaselineares com potenciais singulares ou se anulando no infinito envolvendo não linearidades com um crescimento crítico exponencial

Gilson Mamede de Carvalho *

27/02/2019

Neste trabalho nós estudamos e estabelecemos resultados de existência de solução fraca e de não existência de solução clássica para o problema

$$\begin{cases} -\Delta_N u + V(|x|)|u|^{N-2}u = Q(|x|)h(u), \quad x \in \mathbb{R}^N, \\ u(x) \to 0 \quad \text{se} \quad |x| \to +\infty, \end{cases} \tag{P}$$

onde $N \geq 2$, $\Delta_N u$ denota o N-Laplaciano da função u, $V, Q : (0, +\infty) \to \mathbb{R}$ são potenciais contínuos podendo se anular no infinito ou ter uma singularidade na origem e h uma não linearidade satisfazendo condições adequadas.

Para estudarmos a existência de solução para (P) assumimos V e Q verificando as seguintes hipóteses:

(V_1) $V : (0, +\infty) \to \mathbb{R}$ é um potencial contínuo, $V(r) > 0$ para todo $r > 0$ e existem constantes $a_0 \in \mathbb{R}$ e $a > -N$ tais que

$$0 < \liminf_{r \to 0^+} \frac{V(r)}{r^{a_0}} \quad \text{e} \quad 0 < \liminf_{r \to +\infty} \frac{V(r)}{r^a}.$$

(Q_1) $Q : (0, +\infty) \to \mathbb{R}$ é contínuo, $Q(r) > 0$ para todo $r > 0$ e existem $b_0 > -N$ e $b \leq a$ satisfazendo

$$\limsup_{r \to 0^+} \frac{Q(r)}{r^{b_0}} < +\infty \quad \text{e} \quad \limsup_{r \to +\infty} \frac{Q(r)}{r^b} < +\infty.$$

Além disso, pedimos que a não linearidade $h : \mathbb{R} \to \mathbb{R}$ seja contínua e satisfaça:

(H_1) Existe $\alpha_0 > 0$ tal que

$$\lim_{s \to +\infty} \frac{h(s)}{\mathrm{e}^{\alpha s^{\frac{N}{N-1}}}} = \begin{cases} 0, & \forall\ \alpha > \alpha_0 \\ +\infty, & \forall\ \alpha < \alpha_0. \end{cases}$$

(H_2) $\lim\limits_{s \to 0} \dfrac{h(s)}{s^{N-1}} = 0$;

(H_3) Existe $\mu > N$ tal que

$$0 \leq \mu H(s) := \mu \int_0^s h(t)\,\mathrm{d}t \leq sh(s) \quad \text{para todo} \quad s \in \mathbb{R} \setminus \{0\};$$

(H_4) Existem $\xi > 0$ e $\kappa > N$ tais que

$$H(s) \geq \xi s^\kappa, \quad \forall\ s \geq 0.$$

De posse das condições acima, mostramos o primeiro resultado principal deste trabalho.

Teorema 0.1. *Suponha que (V_1), (Q_1) e (H_1)-(H_4) acontecem. Então, o problema (P) tem uma solução fraca não negativa e não nula. Se assumirmos h satisfazendo também*

*Departamento de Matemtica da Universidade Federal Rural de Pernambuco, email: gilson.carvalho@ufrpe.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

On nonlinear fractional diffusion equations

Giovana Siracusa *

We will talk about existence of solutions for the semilinear fractional diffusion equation

$$\begin{aligned} u_t(t,x) = \int_0^t dg_\alpha(s)\Delta u(t-s,x) + |u(t,x)|^{\rho-1}u(t,x), \text{ in } (0,T)\times\Omega; \\ u(t,x) = 0, \text{ on } (0,T)\times\partial\Omega; \\ u(x,0) = u_0(x), \text{ in } \Omega, \end{aligned}$$

where $g_\alpha(t) = \frac{t^{\alpha-1}}{\Gamma(\alpha)}$, for $\alpha \in (0,1)$, Δ is the Laplace operator, and Ω is a sufficiently smooth domain in $\mathbb{R}^N$. Moreover, we obtain local well-posedness results with initial data in $L^q(\Omega)$, its continuation and the stability of the continued solution.

Joint work with Arlúcio Viana (UFS) and Bruno de Andrade (UFS).

*Departamento de Matemática, Universidade Federal de Sergipe, UFS email: gisiracusa@gmail.com

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Microlocal regularity for Mizohata type differential operators

Gustavo Hoepfner *

In this talk we will show interesting connections between Mizohata type vector fields and microlocal regularity of nonlinear first order PDEs.

Joint work with Renan Medrado (Universidade Federal do Ceará).

*Departamento de Matemática, Universidade Federal de São Carlos, email: hoepfner@dm.ufscar.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Positivity and exact multiplicity results for a sublinear indefinite elliptic equation

Humberto Ramos Quoirin *

We investigate the problem

$$\begin{cases} -\Delta u = a(x)u^q & \text{in} \quad \Omega, \\ \partial_n u = \alpha u & \text{on} \quad \partial\Omega. \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded and smooth domain, $a \in C(\overline{\Omega})$ is **sign-changing**, $0 < q < 1$, and $\alpha \in [-\infty, \infty)$ ($\alpha = -\infty$ means $u = 0$ on $\partial\Omega$). The main feature of this equation is the lack of strong maximum principle structure, so that solutions may vanish in open parts of Ω. For q close to 1 we provide a global description (with respect to α) of the nonnegative solutions set of this problem. In particular, we establish a positivity property and an exact multiplicity result for $\alpha > 0$ small.

Joint work with U. Kaufmann (Córdoba, Argentina) and K. Umezu (Mito, Japan).

*Departamento de Matematicas y C.C. Universidad de Santiago de Chile / FAMAF - Universidad Nacional de Córdoba, Argentina
email: humbertorq@gmail.com

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Null controllability and Stackelberg-Nash strategy for systems of parabolic equations

Islanita Cecília Alcantara de Albuquerque*

This work is the dedicated to solve a multi-objetive control problem in partial differential equations. We have many objectives, possibly conflictive, and a concept of hierarchy must be adopted. The hierarchy is divided between *leaders* and *followers* controls, the *leaders* are responsible for objetives of controllability type, and *followers* controls which intend to be a Nash equilibrium for some cost functionals. The novelty here is that we formulate this problem for systems of parabolic equations, meaning that we have much more variables and naturally much more objectives to accomplish.

Joint work with Maurício Cardoso Santos (Federal University of Paraíba).

*Department of Mathematics, Federal University of Pernambuco, email: islanita@dmat.ufpe.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Shape Optimization Problems for Compressible Navier-Stokes Equations.

Jan Sokolowski *

In the monograph the existence of weak, renormalized global solutions of compressible Navier-Stokes equations for the nonhomogeneous Dirichlet boundary conditions in bounded domains is obtained.

MR2963679 Plotnikov, Pavel ; Sokolowski, Jan . Compressible Navier-Stokes equations. Theory and shape optimization. Instytut Matematyczny Polskiej Akademii Nauk. Monografie Matematyczne (New Series) [Mathematics Institute of the Polish Academy of Sciences. Mathematical Monographs (New Series)], 73. Birkhuser/Springer Basel AG, Basel, 2012. xvi+457 pp. ISBN: 978-3-0348-0366-3

We are interested in shape optimization for systems governed by compressible N-S equations. Domain dependence of solutions to compressible N-S equations is considered in three spatial dimensions. The shape optimization theory is a branch of the general calculus of variations which deals with the shapes of geometric and physical objects instead of parameters and functions. The first global result on domain dependence of solutions to compressible N-S equations is due to Eduard Feireisl, who proved that the set of solutions to compressible N-S equations is compact provided the set of flow domains is compact in the Kuratowski-Mosco topology and their boundaries have "uniformly small" volumes. We have shown in the monograph that the compactness result holds true if the set of flow domains is compact in the Kuratowski-Mosco topology. We have proved also that some cost functionals, such as the drag and the work of hydrodynamical forces, are continuous in this topology. With applications to shape optimization in mind, we have considered the shape differentiability of specific strong solutions and give formulae for the shape derivative of the drag functional.

Our theoretical results are also applied to control theory for distributed parameter systems. We show the existence of an optimal control for a free boundary problem for compressible N-S equations. A one-dimensional free boundary problem of a motion of a heavy piston in a tube filled with viscous gas is considered. The system of governing equations and boundary conditions is derived. The obtained system of differential equations can be regarded as a mathematical model of an exterior combustion engine. The existence of a weak solution to this model is proved. The problem of maximization of the total work of the engine is considered.

Finally, the numerical examples are given in one and two spatial dimensions.

Joint work with Pavel Plotnikov Lavrentyev Institute of Hydrodynamics (Novosibirsk) and Voronej State University (Voronej), Russian Federation.

*Centro de Informatica, Universidade Federal da Paraiba, email: Jan.Sokolowski@univ-lorraine.fr

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Spiked vector solutions of coupled Schrödinger systems with critical exponent and solutions concentrating on spheres

João Marcos B. do Ó *

We consider a class of Gross-Pitaevskii equations (or Bose-Einstein system) defined in an annulus of four-dimensional Euclidean space involving critical growth in sense of Sobolev embedding. In the cooperative case, by using a reduction approach, we prove the existence of semiclassical states by means of a variational approach. The concentration behaviors of these solutions are also studied.

*Departamento de Matemática, Universidade Federal de Brasília, UnB.

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

QUALITATIVE PROPERTIES FOR NON-NEGATIVE SOLUTIONS OF CONFORMALLY-INVARIANT FOURTH-ORDER SYSTEMS

JOÃO HENRIQUE ANDRADE *

We discuss qualitative properties of solutions for strongly coupled critical elliptic systems involving fourth-order operators and critical exponents. We study the system defined in the punctured space with the origin being an isolated singularity. Our results generalize the celebrated study of Caffarelli, Gidas, and Spruck about classification and local behaviour of positive solutions for a class of scalar critical equations related to the classic Yamabe problem. The system we deal enjoys a conformally-invariance property, which we use to show solutions are rotationally invariant around zero. Moreover, we obtain classification (or Liouville-type) results for solutions of the system, either when the origin is a removable or a non-removable singularity.

Joint work with João Marcos do Ó (UnB).

References

[1] C. -S. LIN, A classification of solutions of a conformally invariant fourth order equation in $\mathbf{R}^n$, *Comment. Math. Helv.*, **73**, 206-231, 1998.

[2] P. -L. LIONS, The concentration-compactness principle in the calculus of variations. The locally compact case. I, *Ann. Inst. H. Poincaré Anal. Non Linéaire*, **1**, no. 4, 109-145, 1984.

[3] P. -L. LIONS, The concentration-compactness principle in the calculus of variations. The locally compact case. II, *Ann. Inst. H. Poincaré Anal. Non Linéaire*, **1**, no. 4, 223-283, 1984.

[4] R. L. FRANK AND T. KÖNIG, Classification of positive singular solutions to a nonlinear biharmonic equation with critical exponent, *Anal. PDE*, **2**, no. 4, 1101-1113, 2019.

[5] L. A. CAFFARELLI, B. GIDAS, AND J. SPRUCK, Asymptotic symmetry and local behavior of semilinear elliptic equations with critical Sobolev growth, *Comm. Pure Appl. Math.*, **42**, no. 3, 271-297, 1989.

[6] O. DRUET, E. HEBEY, AND J. VÉTOIS, Bounded stability for strongly coupled critical elliptic systems below the geometric threshold of the conformal Laplacian, *J. Funct. Anal.*, **258**, no. 3, 999-1059, 2010.

[7] R. CAJU, J. M. DO Ó, AND A. SANTOS, Qualitative properties of positive singular solutions to nonlinear elliptic systems with critical growth, it to apper in Ann. Inst. H. Poincaré Anal. Non Linéaire

[8] G. CARISTI AND E. MITIDIERI, Harnack inequality and applications to solutions of biharmonic equations, *Oper. Theory Adv. Appl.*, **168**, 1-26, Birkhäuser, Basel, 2006.

[9] K. K. UHLENBECK AND J. A. VIACLOVSKY, Regularity of weak solutions to critical exponent variational equations, *Math. Res. Lett.*, **7**, no. 5-6, 651-656, 2000.

[10] X. XU, Uniqueness theorem for the entire positive solutions of biharmonic equations in $\mathbf{R}^n$, *Proc. Roy. Soc. Edinburgh Sect. A*, **130**, no. 3, 651-670, 2000.

[11]Y. LI AND L. ZHANG, Liouville-type theorems and Harnack-type inequalities for semilinear elliptic equations, *J. Anal. Math.*, **90**, 27-87, 2003.

[12]C. C. CHEN AND C. S. LIN, Local behavior of singular positive solutions of semilinear elliptic equations with Sobolev exponent, *Duke Math. J.*, **78**, no. 2, 315-334, 1995.

[13]E. H. LIEB, Sharp constants in the Hardy-Littlewood-Sobolev and related inequalities, *Ann. of Math* , **118**, no. 2, 349-374, 1983.

*Department of Mathematics, UFPB email: andradejh@mat.ufpb.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

A Pohozaev identity for a class of elliptic Hamiltonian systems and the Lane-Endem conjecture

Jose Anderson V. Cardoso *

Abstract

In this talk we discuss about nonexistence of solution for a class of Hamiltonian elliptic systems involving Schrödinger equations. In this direction we prove a general nonexistence result that provide, as a particular case, a partial answer for the Lane-Emden conjecture.

Joint work with Diego Ferraz (UFRN) and João Marcos do Ó (Unb).

1 Introduction

The purpose of this work is to study nonexistence of solution result for the following class of hamiltonian elliptic systems

$$\begin{cases} -\Delta v + V(x)v = f(x,u) & \text{in} \quad \mathbb{R}^N, \\ -\Delta u + V(x)u = b(x)g(v) & \text{in} \quad \mathbb{R}^N. \end{cases} \tag{S}$$

To illustrate this difficult when dealing with hamiltonian systems, let us consider $f \sim |u|^{p-2}u$ and $g \sim |v|^{p-2}v$. It was discussed in [2, 3] that the correct notion of subcriticallity with respect to (S) occurs when

$$\frac{1}{p} + \frac{1}{q} > 1 - \frac{2}{N},\ p,q > 1, \tag{$\mathcal{H}_{sub}$}$$

while the true notion of criticality is given for (p,q) such that

$$\frac{1}{p} + \frac{1}{q} = 1 - \frac{2}{N},\ p,q > 1,\ N \geq 3, \tag{$\mathcal{H}_{crit}$}$$

i.e., (p,q) lies on the so called Sobolev *critical hyperbola*.

2 Main Results

Our main result is the key to prove general nonexistence results for System (S) and it asserts that some solutions of (S) must satisfy a certain integral identity. The proof follows by a "local-to-global" cutoff argument.

Teorema 2.1. *Let $V(x) \in C^1(\mathbb{R}^N \setminus \mathcal{O})$, where $\mathcal{O}$ is a finite set, $0 < \beta \leq b(x) \in L^\infty(\mathbb{R}^N) \cap C^1(\mathbb{R}^N)$, $u \in W^{2,q/(q-1)}_{loc}(\mathbb{R}^N)$ and $v \in W^{2,p/(p-1)}_{loc}(\mathbb{R}^N)$ be a pair of strong solution for (S). Suppose that*

$$F(x,u),\ b(x)G(v),\ \sum_{i=1}^N x_i F_{x_i}(x,u),\ \langle \nabla b(x), x\rangle\, G(v),\ V(x)uv,\ \langle \nabla V(x), x\rangle\, uv,\ uf(x,u),\ vb(x)g(v) \in L^1(\mathbb{R}^N)$$

$$\text{and} \qquad 2\int_{\mathbb{R}^N} \langle \nabla u, \nabla v\rangle\, dx = \int_{\mathbb{R}^N} v(b(x)g(v) - V(x)u) + u(f(x,u) - V(x)v)dx. \tag{2.1}$$

*Department of Mathematics, Sergipe Federal University, email: anderson@mat.ufs.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Existence and nonexistence results of critical cases of linearly coupled Choquard systems

José Carlos de Albuquerque *

In this talk we consider existence and nonexistence of ground states for the following class of Choquard coupled systems

$$\begin{cases} -\Delta u + u = (I_\alpha * |u|^p)\,|u|^{p-2}u + \lambda v, & x \in \mathbb{R}^N, \\ -\Delta v + v = (I_\alpha * |v|^q)\,|v|^{q-2}v + \lambda u, & x \in \mathbb{R}^N, \end{cases} \tag{S_α}$$

where $N \geq 3$, $\alpha \in (0, N)$ and $\lambda \in (0, 1)$. Our approach is based on a Nehari manifold technique and Pohozaev identity type.

Joint work with Edcarlos D. Silva (UFG), Maxwell L. Silva (UFG) and Minbo Yang (ZNU).

*Department of Mathematics, Federal University of Pernambuco, email: joserre@gmail.com

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Non null controllability of Stokes equations with memory

José Lucas Ferreira Machado *

Stokes equations have been studied since many years and its understanding is very relevant from the mathematical and physical viewpoint. In this talk, we will consider the Stokes equations in the presence of an integro-differential term (integral in time and differential in space) called *memory term*. We will study the boundary null controllability problem (to steer the flow to the rest at an arbitrarily small time) for the Stokes equations with memory in two and three dimensional cases. Precisely, we will construct explicitly initial conditions such that the null controllability does not hold even if the controls act on the whole boundary. Moreover, we also prove that this negative result holds for distributed controls. Finally, we will present some issues which remain open.

Joint work with Enrique Fernández-Cara (University of Sevilla) and Diego A. Souza (UFPE).

*Department of Mathematics, Federal University of Pernambuco, email: lucasmachado@dmat.ufpe.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Semilinear parabolic equations with asymptotically linear growth

Juliana Fernandes S. Pimentel *

We present some recent work on the existence and behaviour of solutions for a class of semilinear parabolic equations, defined on a bounded smooth n-dimensional domain, and we assume that the nonlinearity is asymptotically linear at infinity. We analyze the behavior of the solutions when the initial data varies in the phase space. We obtain global solutions which may be bounded or blowup in infinite time (grow-up). Our main tools are the comparison principle and variational methods. Particular attention is paid to initial data at high energy level. We use the Nehari manifold to separate the phase space into regions of initial data where uniform boundedness or grow-up behavior of the semiflow may occur.

Joint work with Liliana Maia (Universidade de Brasilia, Brazil).

*Department of Mathematics, Universidade Federal do Rio do Janeiro, email: jfernandes@im.ufrj.br

WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Positive mass theorems in Yamabe-type problems

Levi Lopes de Lima*

Positive mass theorems were first formulated in the physical set- ting of the so-called initial value formulation of General Relativity, but rather surprisingly they turned out to become effective tools in dealing with several deep questions in Geometric Analysis. We illustrate this circle of ideas by providing an overview of the use of positive mass theorems in Yamabe-type problems, with a focus on the case in which the underlying manifold carries a non-empty boundary. We discuss recent developments and mention a few open problems.

*Department of Mathematics, Federal University of Ceará

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

On the semilinear wave equation in a Besov-type setting

Lucas C. F. Ferreira *

We show global-in-time well-posedness and self-similarity for the semilinear wave equation with nonlinearity $f(u) = \pm u^p$ in a time-weighted framework based on the larger family of homogeneous Besov spaces $\dot{B}^{s_q,\infty}_{q,\infty}$ for $q > 2$. As a consequence, in some cases of the power p we cover a larger initial-data class. Our approach relies on dispersive-type estimates and a suitable p-product estimate in Besov spaces.

Joint work with Prof. Jhean E. Pérez-López (UIS, Colombia).

*Department of Mathematics, University of Campinas (Unicamp), email: lcff@ime.unicamp.br. LCFF was supported by FAPESP-SP and CNPq-BR

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Hierarchical exact controllability of semilinear parabolic equations.

Luciano Cipriano da Silva *

Abstract

We present some exact controllability results for parabolic equations in the context of hierarchic control using Stackelberg–Nash strategies. We analyze two cases: in the first one, the main control (the leader) acts in the interior of the domain and the secondary controls (the followers) act on small parts of the boundary; in the second one, we consider a leader acting on the boundary while the followers are of the distributed kind. In both cases, for each leader an associated Nash equilibrium pair is found; then, we obtain a leader that leads the system exactly to a prescribed (but arbitrary) trajectory. We consider linear and semilinear problems.

Joint work with Fágner D. Araruna (Universidade Federal da Paraíba) and Enrique Fernández-Cara (Universidad de Sevilla).

*Instituto Federal do Rio Grande do Norte, luciano.cipriano88@gmail.com

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

On the supercritical gKdV equation

Luiz Gustavo Farah *

In this talk we discuss some results for the supercritical gKdV equation, such as well-posedness, existence of maximizers for Airy-Strichartz inequalities, concentration of blow-up solutions and scattering. These results were obtained in collaboration with Ademir Pastor (UNICAMP-Brazil), Brian Pigott (Wofford College-USA), Felipe Linares (IMPA-Brazil), Henrique Versieux (UFMG-Brazil), Nicola Visciglia (UNIPI-Italy). The author was partially supported by CNPq, CAPES and FAPEMIG-Brazil.

References

[1] L. G. Farah, F. Linares, A. Pastor and N. Visciglia, Large data scattering for the defocusing supercritical generalized KdV equation, *Comm. Partial Differential Equations*, **43**, 118-157, 2018.

[2] L. G. Farah and A. Pastor, On well-posedness and wave operator for the gKdV equation, *Bull. Sci. Math.*, **137**, 229–241, 2013.

[3] L. G. Farah and B. Pigott, Nonlinear profile decomposition and the concentration phenomenon for supercritical generalized KdV equations, *Indiana Univ. Math. J.*, **67**, 1857-1892, 2018.

[4] L. G. Farah and H. Versieux, On the lack of compactness and existence of maximizers for some Airy-Strichartz inequalities., *Differential and Integral Equations*, **31**, 27–56, 2018.

*Department of Mathematics, Universidade Federal de Minas Gerais, email: lgfarah@gmail.com

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Sharp regularity estimates for quasilinear evolution equations

Marcelo Dario dos Santos Amaral *

We establish sharp geometric $C^{1+\alpha}$ regularity estimates for bounded weak solutions of evolution equations of $p-$Laplacian type. Our approach is based on geometric tangential methods, and makes use of a systematic oscillation mechanism combined with an adjusted intrinsic scaling argument.

*Universidade da Integração Internacional da Lusofonia Afro-Brasileira, Brasil

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Minimizing movement for a fractional porous medium equation in a periodic setting

Matheus Correia dos Santos *

We consider a fractional porous medium equation that extends the classical porous medium and fractional heat equations. The flow is studied in the space of periodic probability measures endowed with a non-local transportation distance constructed in the spirit of the Benamou-Brenier formula. For initial periodic probability measures, we show the existence of absolutely continuous curves that are generalized minimizing movements associated to Rényi entropy.

Joint work with Lucas C.F.Ferreira (Unicamp-Brazil) and Julio C. Valencia-Guevara (Universidad Catlica San Pablo-Peru).

*Department of Mathematics and Statistics, Universidade Federal do Rio Grande do Sul, email: matheus.santos@ufrgs.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Nonlinear persistence results for eigenvalues and eigenvectors of Fredholm operators with set-valued perturbations

Pierluigi Benevieri *

We consider the nonlinear multivaled parametrized problem

$$\begin{cases} Lx - \lambda Cx \in \varepsilon\phi(x) \\ x \in \partial\Omega, \end{cases}$$

where, given two real Banach spaces E and F,

i) $L, C : E \to F$ are bounded linear operators, with L Fredholm of index zero,

ii) Ω is an open subset of E with $0 \in \Omega$,

iii) $\phi : \overline{\Omega} \multimap F$ is a locally compact, upper semi-continuous, set-valued map and such that $\phi(x)$ is an R_δ-set for any $x \in \overline{\Omega}$, where a subset A of a metric space Y is called an *R_δ-set* if $A = \cap_{n=1}^{\infty} A_n$, and (A_n) is a sequence of compact, contractible subsets of Y. (For example, convex sets and contractible sets are R_δ-sets),

iv) ε and λ are real parameters.

We prove a local bifurcation result regarding the set Σ of the *solutions* $(x, \varepsilon, \lambda) \in \partial\Omega \times \mathbb{R} \times \mathbb{R}$ of this problem. Namely, under suitable transversality assumptions on L, we prove that there exists $\overline{x} \in \partial\Omega \cap \operatorname{Ker} L$ such that any neighborhood of $(\overline{x}, 0, 0)$ in $(\partial\Omega \cap \operatorname{Ker} L) \times \mathbb{R} \times \mathbb{R}$ contains solutions $(x, \varepsilon, \lambda)$, with $(\varepsilon, \lambda) \neq (0, 0)$. The approach is topological, based on a extension of the Leray–Schauder degree to set-valued maps. We finally give an application to a differential inclusion of the second order, coupled with Neumann boundary conditions and an integral constraint:

$$\begin{cases} u'' + u' - \lambda u + \varepsilon\phi(u) \ni 0 \text{ in } [0,1] \\ u'(0) = u'(1) = 0 \\ \|u\|_{L^1} = 1. \end{cases}$$

Our work is inspired by a paper of R. Chiappinelli concerning the local persistence property of the unit eigenvectors of perturbed self-adjoint operators in a real Hilbert space. This is a joint work with A. Iannizzotto (Università di Cagliari).

*Instituto de Matemática e Estatística, Universidade de São Paulo, email: pluigi@ime.usp.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

On Some Geometric Inverse Problems for Elliptic and Parabolic Systems

Raul Kazan da C. Araujo *

Inverse Problems are concerned with determining causes for a desired or observed effect and then most of the inverse problems in practice are ill-posed. In this talk, we present some results for the inverse problem of the identification of a non-reacting single rigid body immersed in a chemical product, sensible to temperature, governed by a system of elliptic equations. First, we establish a uniqueness result. Then, we show the way the observation depends on perturbations of the non-reacting rigid body and we deduce some consequences related to the reconstruction for the inverse problem. The main techniques are related to (local) Carleman estimates and differentiation with respect to the domain. Finally, we will present some issues which remain open.

Joint work with Enrique Fernández-Cara (University of Sevilla) and Diego A. Souza (UFPE).

*Department of Mathematics, UFPE, email: raulkkazan.1@gmail.com

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Regularity of stable solutions to quasilinear elliptic equations on Riemannian models

Rodrigo Clemente *

We investigate the regularity of semi-stable, radially symmetric, and decreasing solutions for a class of quasilinear reaction-diffusion equations in the inhomogeneous context of Riemannian manifolds. We prove uniform boundedness, Lebesgue and Sobolev estimates for this class of solutions for equations involving the p-Laplace Beltrami operator and locally Lipschitz non-linearity. We emphasize that our results do not depend on the boundary conditions and the specific form of the non-linearities and metric. Moreover, as an application, we establish regularity of the extremal solutions for equations involving the p-Laplace Beltrami operator with zero Dirichlet boundary conditions.

Joint work with João Marcos do Ó (UnB).

*Department of Mathematics, Universidade Federal Rural de Pernambuco, email: rodrigo.clemente@ufrpe.br

WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

Existence of steady periodic water waves

Silvia Sastre*

In this talk, we apply bifurcation theory to prove the existence of small and large- amplitude steady periodic water waves, which propagate over a flat bed with a specified fixed mean-depth, and where the underlying flow has a discontinuous vorticity distribution.

Joint work with David Henry (UCC, Ireland) and Calin I. Martin (University of Vienna).

*Department of Mathematics, Federal University of Pernambuco

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

UNIFORM NULL CONTROLLABILITY FOR FAST DIFFUSION PARABOLIC SYSTEMS WITH FIRST ORDER COUPLING TERMS

Thiago Y. Tanaka *

In this work, we investigate a uniform null controllability result for coupled linear parablic systems which one of the equations is approaching an elliptic one. More precisely, Given $\epsilon > 0$, $\{a_{i,j}\}_{i,j=1}^2$ in $L^\infty(Q_T;\mathbb{R})$ and $\{A_{i,j}\}_{i,j=1}^2$ in $L^\infty(Q_T;\mathbb{R}^N)$, consider the following system of parabolic equations

$$\begin{cases} y_t^\epsilon - \Delta y^\epsilon = a_{1,1}y^\epsilon + A_{1,1}\cdot\nabla y^\epsilon + a_{1,2}z^\epsilon + A_{1,2}\cdot\nabla z^\epsilon + f_1^\epsilon 1_{\omega_1}, & \text{in } Q_T, \\ \epsilon z_t^\epsilon - \Delta z^\epsilon = a_{2,1}z^\epsilon + A_{2,1}\cdot\nabla z^\epsilon + a_{2,2}y^\epsilon + A_{2,2}\cdot\nabla y^\epsilon + f_2^\epsilon 1_{\omega_2}, & \text{in } Q_T, \\ y^\epsilon = z^\epsilon = 0, & \text{on } \Sigma_T, \\ y^\epsilon|_{t=0} = y_0; z^\epsilon|_{t=0} = z_0, & \text{in } \Omega. \end{cases} \tag{0.1}$$

It is well known that for any $\epsilon > 0$ fixed, system (0.1) is null controllable in the sense that there exists functions $f_1^\epsilon, f_2^\epsilon \in L^2(Q_T)$ such that $y^\epsilon(\cdot,T) = z^\epsilon(\cdot,T) = 0$. Here we want to know whether it is possible to find controls $(f_1^\epsilon, f_2^\epsilon)$ which are uniform concerning ϵ in the sense that its costs is uniformly bounded with respect to this parameter. The reason for this is that in many applications , system (0.1) is approximated by systems of the form

$$\begin{cases} y_t - \Delta y = a_{1,1}y + A_{1,1}\cdot\nabla y + a_{1,2}z + A_{1,2}\cdot\nabla z + f_1 1_{\omega_1}, & \text{in } Q_T \\ -\Delta z = a_{2,1}z + A_{2,1}\cdot\nabla z + a_{2,2}y + A_{2,2}\cdot\nabla y + f_2 1_{\omega_2}, & \text{in } Q_T, \\ y = z = 0, & \text{on } \Sigma_T, \\ y|_{t=0} = y_0, z|_{t=0} = z_0 & \text{in } \Omega. \end{cases} \tag{0.2}$$

This can be seen, for instance, in aggregation phenomena and in chemical reactions with two different concentrations. It is important to mention that, in [1], *Chaves-Silva et al* already proved the existence of uniform controls for systems like (0.1) but for zero order couplings, that is, assuming that $A_{1,1} = A_{1,2} = A_{2,1} = A_{2,2} = 0$. In this work we extend their results for the case of first order couplings. Moreover, the proofs contained here are much simpler than the ones in [1] which can be of use when dealing with more complex problems.

In fact, we extend the results of *Chaves-Silva et al.* to the case of systems of first order coupling terms. The strategy is to prove a suitable observability inequality with observability constant uniformly bounded with respect to the diffusion parameter. As a consequence, when the parameter goes to zero, we obtain a null controllability result for a parabolic-elliptic system with first order coupling coefficients. In particular, our proof simplifies the one given in *Chaves-Silva et al.* in the case of zero order coupling terms.

Joint work with Maurcio C. Santos (UFPB), Felipe W. Chaves-Silva (UFPB) and Diego A. Souza (UFPE).

References

[1] f. w. chaves-silva, s. guerrero, j.-p. puel, *Controllability of fast diffusion coupled parabolic systems*, Mathematical Control and Related Fields, 4 (4)(2014), 465 - 479.

*Department of Mathematics, Universidade Federal Rural de Pernambuco , email: thiago.tanaka@ufrpe.br

VIII WENLU - Workshop on Nonlinear Partial Differential Equations and Geometric Analysis
UFPB - Universidade Federal da Paraíba
João Pessoa - Brazil, March 20-22, 2019

On the Morse index of radial solutions of the Hénon equation in dimension two

Wendel Leite da Silva *

We consider the Hénon type problem

$$\begin{cases} -\Delta u = |x|^{\alpha} f(u) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^2$ is either a ball or an annulus centered at the origin, $\alpha \geq 0$ and $f \in C^1(\mathbb{R})$. We obtained results about the computation of the Morse index of both positive and nodal radial solutions. The contribution of this work is twofold. Firstly, fixed the number of nodal sets $n \geq 1$ of the solution $u_{\alpha,n}$, we prove that the Morse index $m(u_{\alpha,n})$ is monotone non-decreasing with respect to α. Secondly, we provide a lower bound for the Morse indices $m(u_{\alpha,n})$, which shows that $m(u_{\alpha,n}) \to +\infty$ as $\alpha \to +\infty$.

Joint work with Ederson Moreira dos Santos (ICMC-USP).

*Instituto de Ciências Matemáticas e de Computação, Universidade de São Paulo, email: wendelleite@usp.br