Some class of Schrödinger equations involving laplacian and p-laplacian operators with vanishing potentials *

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Abstract

In this paper we study the existence of weak positive solutions for the following class of quasilinear Schrödinger equations

$$-\Delta_p u - \Delta u + V(x)u = f(u) \quad \text{in} \quad \mathbb{R}^N,$$

where f satisfies some "mountain–pass" type assumptions and V is a nonnegative continuous function. We give a special attention to the case when V may eventually vanish at infinity. Our arguments are based on penalization techniques, variational methods and Moser iteration scheme.

Key words. Quasilinear problem, Variational Methods, Mountain–pass theorem, Standing wave solution

1 Introduction

This paper concerns the existence of weak solutions of the following nonlinear field equation

$$\begin{cases} -\Delta_p u - \Delta u + V(x)u = f(u) & \text{in} \quad \mathbb{R}^N, \\ u > 0 & \text{in} \quad \mathbb{R}^N, \\ u \in \ D_r^{1,2}(\mathbb{R}^N) \cap D_r^{1,p}(\mathbb{R}^N), \end{cases} \tag{P}$$

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where $2 < p < N$, $\Delta_p u := \text{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplace operator and $p^* = Np/(N-p)$ is the critical Sobolev exponent. We assume that the nonlinearity $f : \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying:

(f_1) There exists $\gamma \in (1, p^* - 1]$ such that $\lim_{s \to 0^+} \dfrac{f(s)}{s^\gamma} = 0$;

(f_2) $\lim_{s \to +\infty} \dfrac{f(s)}{s^{p^*-1}} = 0$;

(f_3) (Ambrosetti-Rabinowitz condition) there exists $p < \theta \le p^*$ such that

$$0 < \theta F(s) \le s f(s) \quad \text{for all } s > 0, \quad \text{where} \quad F(s) \equiv \int_0^s f(t)dt,$$

and the potential $V : \mathbb{R}^N \to \mathbb{R}$ satisfies the following conditions:

(V_1) $\liminf_{|x| \to \infty} |x|^\alpha V(x) > 0$, where $\alpha = (N-2)(\gamma-1)/2$;

(V_2) $V \in C(\mathbb{R}^N, [0, \infty))$ is a radial function.

The main result of this paper is presented below:

Theorem 1.1 *Assume* $(f_1) - (f_3)$ *and* $(V_1) - (V_2)$. *Then there exists a positive solution for Problem* (P).

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