Existence of multi-bump solutions for a class of elliptic problems involving the biharmonic operator *

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Abstract

Using variational methods, we establish existence of multi-bump solutions for the following class of problems

$$\begin{cases} \Delta^2 u + (\lambda V(x) + 1)u = f(u), & \text{in} \quad \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N), \end{cases}$$

where $N \geq 1$, Δ^2 is the biharmonic operator, f is a continuous function with subcritical growth, $V : \mathbb{R}^N \to \mathbb{R}$ is a continuous function verifying some conditions and $\lambda > 0$ is a real constant large enough.

1 Introduction

In this work, based in [5] and [1], we are concerned with the existence of multi-bump solutions for the following class of problems

$$\begin{cases} \Delta^2 u + (\lambda V(x) + 1)u &= f(u), \quad \text{in} \quad \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N); \end{cases} \tag{1.1}$$

where $N \geq 1$, Δ^2 denotes the biharmonic operator, $\lambda > 0$ is a positive parameter and $f : \mathbb{R} \to \mathbb{R}$ is a C^1, subcritical function and the potential $V : \mathbb{R}^N \to \mathbb{R}$, we assume the following assumptions :

(V_1) $V(x) \geq 0, \ \forall \ x \in \mathbb{R}^N$;

(V_2) $\Omega = intV^{-1}(\{0\})$ is a non-empty bounded open set with smooth boundary $\partial\Omega$. Moreover, Ω has k connected components, more precisely,

- $\Omega = \bigcup_{j=1}^{k} \Omega_j$;
- $dist(\Omega_i, \Omega_j) > 0, \ i \neq j$.

(V_3) There is $M_0 > 0$ such that $|\{x \in \mathbb{R}^N; V(x) \leq M_0\}| < +\infty$.

In general, the works that are proposed to investigate the existence of multi-bump solutions need of the Penalization Method [4]. In the our case, it is not clear that the method developed in [4] can be used for our problem, because we are working with biharmonic operator. To overcome this difficulty, we have developed a new approach to get multi-bump avoiding the penalization on the nonlinearity. Our inspiration comes from an approach used in Bartsch & Wang [2, 3]. Here, we modify the sets where we will apply the Deformation Lemma.

*Article submitted to VII WENLU

†C. O. Alves was partially supported by CNPq/Brazil 301807/2013-2 and INCT-MAT, coalves@mat.ufcg.edu.br

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2 Main Results

The main results of this paper is the following:

Theorem 2.1. *Suppose that* $(f_1)-(f_4)$ *and* $(V_1)-(V_3)$ *hold. Then, for each non-empty subset* $\Gamma \subset \{1,\cdots,k\}$ *and* $\varepsilon > 0$ *fixed, there is a* $\lambda^* = \lambda^*(\varepsilon) > 0$ *such that, (1.1) possesses a solution* u_λ*, for* $\lambda \geq \lambda^*$*, satisfying:*

$$\left|\frac{1}{2}\int_{\Omega_j}\left[|\Delta u_\lambda|^2 + (\lambda V(x)+1)|u_\lambda|^2\right]dx - \int_{\Omega_j} F(u_\lambda)dx - c_j\right| < \varepsilon, \forall j \in \Gamma$$

and

$$\int_{\mathbb{R}^N\setminus\Omega_\Gamma}\left[|\Delta u_\lambda|^2 + |u_\lambda|^2\right]dx < \varepsilon,$$

where $\Omega_\Gamma = \cup_{j\in\Gamma}\Omega_j$ *and* c_j *is the minimax level of the energy functional related to the problem:*

$$\begin{cases} \Delta^2 u + u = f(u), & in \quad \Omega_j \\ u = \frac{\partial u}{\partial \eta} = 0, & on \quad \partial\Omega_j. \end{cases} \tag{2.2}$$

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