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Sharp isoperimetric inequalities for small volumes in complete noncompact Riemannian manifolds of bounded geometry involving the scalar curvature

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We provide an isoperimetric comparison theorem for small volumes in a n-dimensional Riemannian manifold (M^n, g) with C^3 bounded geometry in a suitable sense involving the scalar curvature function. Under C^3 bounds of the geometry, if the supremum of scalar curvature function $S_g < n(n-1)k_0$ for some $k_0 \in \mathbb{R}$, then for small volumes the isoperimetric profile of (M^n, g) is less then or equal to the isoperimetric profile of the complete simply connected space form of constant sectional curvature k_0. This work generalizes Theorem 2 of [1] in which the same result was proved in the case where (M^n, g) is assumed to be compact. As a consequence of our result we give an asymptotic expansion in Puiseux series up to the second nontrivial term of the isoperimetric profile function for small volumes, generalizing our earlier asymptotic expansion [2]. Finally, as a corollary of our isoperimetric comparison result, it is shown that for small volumes the Aubin-Cartan-Hadamard's Conjecture is true in any dimension n in the special case of manifolds with C^3 bounded geometry, and $S_g < n(n-1)k_0$. Two different intrinsic proofs of the fact that an isoperimetric region of small volume is of small diameter. The first under the assumption of mild bounded geometry, i.e., positive injectivity radius and Ricci curvature bounded below. The second assuming the existence of an upper bound of the sectional curvature, positive injectivity radius, and a lower bound of the Ricci curvature.

Joint work with Luis Eduardo Osorio Acevedo (UFRJ-Universidade Federal do Rio de Janeiro).

References

[1] O. Druet, *Sharp local isoperimetric inequalities in- volving the scalar curvature*, Proc. Amer. Math. Soc., 130(8):2351?2361 (electronic), 2002.

[2] Stefano Nardulli, The isoperimetric profile of a noncompact Riemannian manifold for small volumes, *Calc. Var. Partial Differential Equations*, 49(1-2):173–195, 2014.

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