Multiple positive solutions to a semilinear Dirichlet problem in an exterior domain

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We will present a recent work where we establish the existence of multiple positive solutions to the semilinear Dirichlet problem

$$-\Delta u + \lambda u = f(u), \qquad u \in H^1_0(\Omega_R),$$

where $\Omega_R := \{x \in \mathbb{R}^N : |u| > R\}$ with R > 0, N = 2 or $N \ge 4$, $\lambda > 0$, and the nonlinearity f is either asymptotically linear, or superlinear and subcritical at infinity. We show that the number of positive nonradial solutions becomes arbitrarily large as $R \to \infty$.

When Ω is the complement of a ball, the problem is known to have a positive radial solution; see [1] and [2]. Thus, it is natural to ask whether the solution found in [3] coincides with the radial one or not. We shall see that it does not, if R is sufficiently large. Moreover, we will show that the number of positive nonradial solutions to this problem becomes arbitrarily large, as $R \to \infty$, when $N \neq 3$.

Joint work with Mónica Clapp (Universidad Nacional Autnoma de Mxico) and Benedetta Pellacci (Universit di Napoli Parthenope).

References

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