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A nonexistence result for a nonhomogeneous asymptotically linear equation

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In this talk we consider the following Schrödinger equation in $\mathbb{R}^N$, for $N \geq 3$ and $\lambda > 0$:

$$-\Delta u + \lambda u = \frac{u^3}{1 + s(x)u^2}, \tag{0.1}$$

where $s : \mathbb{R}^N \to \mathbb{R}$ is given by $s(x) = \frac{1}{1+|x|^2} + s_\infty$, with $s_\infty > 0$.

The functional associated with (0.1) is given by $I : H^1(\mathbb{R}^N) \to \mathbb{R}$,

$$I(u) = \frac{1}{2}\int_{\mathbb{R}^N} |\nabla u|^2 + \lambda u^2 dx - \int_{\mathbb{R}^N} \frac{u^2}{2s(x)} - \frac{1}{2s^2(x)} \ln(1 + s(x)u^2) dx.$$

Since $\lim_{|x|\to\infty} s(x) = s_\infty$, we can define the limit problem

$$-\Delta u + \lambda u = \frac{u^3}{1 + s_\infty u^2}, \tag{0.2}$$

and the functional

$$I_\infty(u) = \frac{1}{2}\int_{\mathbb{R}^N} |\nabla u|^2 + \lambda u^2 dx - \int_{\mathbb{R}^N} \frac{u^2}{2s_\infty} - \frac{1}{2s_\infty^2} \ln(1 + s_\infty u^2) dx$$

associated with this limit problem. It is easy to verify that the functionals I and I_∞ satisfy the unusual relation $I_\infty(u) < I(u), \forall u \in H^1(\mathbb{R}^N) \setminus \{0\}$.

Considering the Pohozaev identity related with (0.1), given by

$$\frac{N-2}{2}\int_{\mathbb{R}^N} |\nabla u|^2 dx = N\int_{\mathbb{R}^N} \frac{u^2}{2s(x)} - \frac{1}{2s^2(x)} \ln(1 + s(x)u^2) - \frac{\lambda u^2}{2} dx \tag{0.3}$$

we can define the Pohozaev manifold $\mathcal{P}$ by

$$\mathcal{P} = \left\{ u \in H^1(\mathbb{R}^N) \setminus \{0\}; u \quad satisfies \ (0.3) \right\}.$$

Working with projections over the Pohozaev manifold $\mathcal{P}$, and the Pohozaev manifold $\mathcal{P}_\infty$, associated with the limit problem (0.2) we can provide a characterization for the Mountain Pass level of the functional I,

$$c := \min_{\gamma \in \Gamma} \max_{0 \leq t \leq 1} I(\gamma(t)).$$

where

$$\Gamma = \left\{ \gamma \in C([0,1], H^1(\mathbb{R}^N)) | \gamma(0) = 0, I(\gamma(1)) < 0 \right\}.$$

Theorem 1: Considering $p = \inf_{u \in \mathcal{P}} I(u)$ and c_∞, the Mountain Pass level for the functional I_∞, we have that

$$c = p = c_\infty > 0.$$

Theorem 2: The infimum $p = \inf_{u \in \mathcal{P}} I(u)$ is not a critical level for the functional I. In particular, the infimum p is not achieved.

With these results, we conclude that we need raise the energy level of the functional I in order to find a solution for equation (0.1).

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