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Liouville theorems for radial solutions of semilinear elliptic equations

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Abstract

In this work we obtain some new Liouville theorems for positive, radially symmetric solutions of the equation

$$-\Delta u = f(u) \qquad \text{in } \mathbb{R}^N$$

where f is a continuous function in $[0, +\infty)$ which is positive in $(0, \infty)$. Our methods adapt to cover more general problems, where the nonlinearity is multiplied by some radially symmetric weights and/or the Laplacian is replaced by the p-Laplacian, $1 < p < N$. Some results for related elliptic systems are also obtained.

Joint work with A. Quaas (Departamento de Matemática, Universidad Técnica Federico Santa María) and J. Garca-Melin (Departamento de Análisis Matemático, Universidad de La Laguna).

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