SHARP REGULARITY FOR THE DEGENERATE DOUBLY NONLINEAR PARABOLIC EQUATION

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ABSTRACT. The aim of this paper is to obtain sharp regularity estimates for locally bounded solutions of the degenerate doubly nonlinear equation

$$u_t - \operatorname{div}(mu^{m-1}|\nabla u|^{p-2}\nabla u) = f,$$

where $m > 1$, $p > 2$ and $f \in L^{q,r}$. More precisely, we show that solutions are locally of class $C^{0,\beta}$, where β depends explicitly only on the optimal Hölder exponent for solutions of the homogeneous case, the integrability of f, the constants p, m and the space dimension n.

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