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Minimal time issues for Grushin type equations

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The goal of this talk is to present several sharp results on the minimal time required for observability of several Grushin-type equations. Namely, it is by now well-known that Grushin-type equations are degenerate parabolic equations for which some geometric conditions are needed to get observability properties, contrarily to the usual parabolic equations. Our results concern the Grushin operator $\partial_t - \Delta_{xx} - |x|^2\Delta_{yy}$ observed from the whole boundary in the multi-dimensional setting, from one lateral boundary in the one-dimensional setting, including some generalized version of the form $\partial_t - \partial_{xx} - (q(x))^2\partial_{yy}$ for suitable functions q ($q(x) \simeq x$ in a sense to be made precise), and the Heisenberg operator $\partial_t - \partial_{xx} - (x\partial_z + \partial_y)^2$ observed from one lateral boundary. In all these cases, our approach strongly relies on the analysis of the family of one-dimensional equations obtained by using the Fourier expansion of the equations in the y (or (y, z)) variables, and in particular the asymptotic of the cost of observability in the Fourier parameters. Combining these estimates with results on the rate of dissipation of each of these equations, we obtain observability estimates in suitably large times. We shall then explain that the times we obtain to get observability are optimal in several cases using Agmon type estimates.

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