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Existence of strong traveling waves for a combustion model in a porous medium

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In this work we consider the existence of combustion fronts propagating through a one dimensional porous media consisting of oil and gas [1]. Such fronts are modeled as traveling waves of the reaction-difusion-convection PDE system

$$\begin{cases} s_t + f_x = -(hs_x)_x \\ \big((\alpha - s)\theta - \eta\epsilon s\big)_t + \big((\beta - f)\theta - \eta\epsilon f\big)_x = \big((\theta + \eta\epsilon)hs_x + \gamma\theta_x\big)_x \\ (\epsilon s)_t + (\epsilon f)_x = -(\epsilon h s_x)_x + \zeta s q\,, \end{cases}$$

where x is position, t is time, $s(x,t)$ is the gas saturation, $\theta(x,t)$ the temperature, $\epsilon(x,t)$ the fraction of burned oxygen, f is the flux function given by $f(s,\theta) = \frac{s^2}{s^2+(0.1+\theta)(1-s)^2}$, $h(s,\theta)$ is some negative function, q is the reaction rate given by

$$q(\epsilon,\theta) = \begin{cases} (1-\epsilon)A_a e^{-\frac{E}{(\theta-\theta_0)}}, \quad if \quad \theta > \theta_0, \\ 0, \quad if \quad 0 < \theta \leq \theta_0\,, \end{cases}$$

and α, β, γ, η, ζ, E, θ_0, A_a are nonnegative constants depending on the physical properties of the porous medium and fluids.

The existence of the traveling waves is reduced to the study of the existence of heteroclinic orbits connecting a hyperbolic to a nonhyperbolic equilibria of a 3×3 ODE system in which one equilibrium represents the composition of the porous medium before the passage of the combustion front and the other the composition after it. We are interested in traveling waves that are *strong* in the sense that the orbits approach the nonhyperbolic equilibrium by its stable manifold and not along its center direction. We determine two disjoint and closed intervals for θ such that each endpoint of each interval defines a strong traveling wave (combustion front) speed. The results are obtained using mainly the geometric singular perturbation theory [2] and invariant regions [3].

Joint work with J. C. da Mota (UFG).

References

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[3] J. C. da Mota, A. J. de Souza, *Multiple Traveling Waves for Dry Forward Combustion Through a Porous Medium* , SIAM Journal on Applied Mathematics, (2018), To appear.

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