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Symmetry results for positive solutions of fully nonlinear nonlocal operators

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In this work, we study symmetry property of positive solutions for Fully Nonlinear integro-differential equations

$$\begin{cases} \mathfrak{M}^-(u) = f(u) & \text{in } B_1(0) \\ u = 0 & \text{in } \quad \mathbb{R}^N \setminus B_1(0), \end{cases}$$

where $N \geq 1$, $x \in B_1(0) = \{x \in \mathbb{R}^N : |x| < 1\}$, the operator $\mathfrak{M}^-$ is a nonlocal extremal Pucci operator type. We would like of the establish the nonlocal counterpart of the result of daLio and Sirakov in [2], as well as [3] is a nonlocal counterpart of the [4]. The extremal Pucci operator considered here are motivated by paper [1] due to Caffarelli and Silvestre, and are defined as follow

$$\mathfrak{M}^-_{\mathcal{S}}(u) = \ inf_{L\in\mathcal{S}} L(u)$$

where $\mathcal{S}$ is a class of integro-differential operators of the form

$$L(u)(x) = \ int_{\mathbb{R}^N}(u(y) - u(x))K(|x-y|)dy,$$

satisfying

(i) the potential $K(y) = K(|y|)$ is radially symmetric and decreasing

(ii) there exists $0 < \mathbf{c} \leq \ \mathbf{C}$ such that

$$\frac{\mathbf{c}}{|y|^{N+2s}} \leq K(y) \leq \frac{\mathbf{C}}{|y|^{N+2s}}, \tag{0.1}$$

with $0 < s < 1$ and

(iii) the integral

$$\int_{R^N} \frac{|y|^2}{1+|y|^2} K(y)dy \qquad \text{is finite.}$$

In our proof we make use a Maximum Principle for small domain to start the moving planes to obtain the symmetry results of positive solutions.

Joint work with Disson Soares dos Prazeres (Federal University of Sergipe).

References

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