

WEIGHTED TRUDINGER-MOSER INEQUALITIES AND ASSOCIATED LIOUVILLE TYPE EQUATIONS

MARTA CALANCHI *

We discuss some Trudinger–Moser inequalities with weighted Sobolev norms. Suitable logarithmic weights in these norms allow an improvement in the maximal growth for integrability, when one restricts to radial functions.

The main results concern the application of these inequalities to the existence of solutions for certain mean-field equations of Liouville-type. Sharp critical thresholds are found such that for parameters below these thresholds the corresponding functionals are coercive and hence solutions are obtained as global minima of these functionals. In the critical cases the functionals are no longer coercive and solutions may not exist.

We also discuss a limiting case, in which the allowed growth is of double exponential type. Surprisingly, we are able to show that in this case a local minimum persists to exist for critical and also for slightly supercritical parameters. This allows to obtain the existence of a second (mountain-pass) solution, for almost all slightly supercritical parameters, using the Struwe monotonicity trick. This result is in contrast to the non-weighted case, where positive solutions do not exist (in star-shaped domains) in the critical and supercritical case.

Joint work with Eugenio Massa, (Universidade de São Paulo) and Bernhard Ruf (Università degli Studi di Milano).

*Dipartimento di Matematica, Università degli Studi di Milano email: marta.calanchi@unimi.it