VII Workshop in Nonlinear PDE's and Geometric Analysis

February 20-24, 2018
João Pessoa – Paraíba - Brazil

Book of Abstracts

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Universidade Federal da Paraíba

VII WORKSHOP IN NONLINEAR PDE'S AND GEOMETRIC ANALYSIS

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ABOUT THE WORKSHOP

The Workshop in Nonlinear Partial Differential Equations and Geometric Analysis is by now a familiar conference among researchers in the areas. Originally called Workshop em Equações Não Lineares da UFPB (WENLU), the event now includes participants from several countries and therefore a redesign in its structure (and even the name) was required. This is the 7th edition and it will be held in Estação das Artes, a gorgeous convention complex located in one of the most touristic areas of João Pessoa - Brazil, which was kindly provided by the Municipal Government of João Pessoa - PMJP.

The main objective of this event is to bring together researchers to disseminate their work in progress, allowing an exchange of ideas among experts and exposing students and recent PhDs to research themes that have been developed in several institutions from Brazil and abroad.

The organizing committee of the *VII Workshop in Nonlinear PDE's and Geometric Analysis* wishes to express their gratitude to the institutions that supported and made possible the realization of this event: UFPB, CAPES, CNPq, FAPESP, INCTMat and PMJP. Thanks also to all the attendees, as well as to the employees for their enthusiasm and effort, which were important to the development of all the event activities.

The organizing committee

SUMMARY

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Plenary Speaks

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A variational principle for problems in partial differential equations and Analysis

Abbas Moameni *

In a wide range of mathematical problems the existence of a solution is equivalent to the existence of a fixed point for a suitable map or a critical point for an appropriate variational or hemi-variational problem. In particular, in real life applications we are interested in finding such solutions which possesses certain properties. The existence theory is therefore of paramount importance in several areas of mathematics and other sciences. In this paper we shall provide a variational principle that allows us to solve problems of the general form $0 \in \mathcal{F}(u)$, for a possibly multi-valued map $\mathcal{F}$ on a given convex set K. This variational principle has many applications in partial differential equations while unifies and generalizes several results in nonlinear Analysis such as the fixed point theory, critical point theory on convex sets and the principle of symmetric criticality.

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Uniform boundedness of positive solutions of the Lane-Emden equation in dimension two

Boyan Sirakov *

We prove that positive solutions of the Lane-Emden equation in a two-dimensional smooth bounded domain are uniformly bounded for all large exponents. A consequence is an integral bound which implies sharp asymptotic for such solutions. Joint work with Nikola Kamburov.

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Differential operators as functions

Carlos Tomei *

Nonlinear objects are complicated, but the insight on certain classes of differential equations provided by geometric arguments, sometimes visual, sometimes of a computational nature, is surprising (think of the Ambrosetti-Prodi theorem). There is more to learn: the intrinsic difficulty of the objects being studied. We present examples among functions in finite dimensional spaces, ordinary and partial differential operators and their discretizations, and more.

Joint work with H. Bueno (UFMG), D.Burghelea (Ohio U.), J. Cal Neto (UNIRIO), M.Calanchi (U. Milano), O. Kaminski (UFES), I.Malta (PUC-Rio), N.Saldanha (PUC-Rio), B.Sirakov (PUC-Rio), E. Teles (CEFET), A.Zaccur (PUC-Rio).

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Bifurcation properties for a class of fractional Laplacian equations in $\mathbb{R}^N$

Claudianor O. Alves *

In this conference we will talk about the existence of solution for the following class of nonlocal problems

$$\begin{cases} (-\Delta)^s u = \lambda f(x)(u + h(u)), \text{ in } \mathbb{R}^N, \\ u(x) > 0, \quad \forall x \in \mathbb{R}^N, \\ \lim_{|x|\to\infty} u(x) = 0, \end{cases} \tag{P}$$

where $N > 2s$, $s \in (0,1)$, $\lambda > 0$, $f : \mathbb{R}^N \to \mathbb{R}$ is a positive continuous function, $h : \mathbb{R} \to \mathbb{R}$ is a bounded continuous function and $(-\Delta)^s u$ is the fractional Laplacian. The main tools used are Leray-Shauder degree theory and Global Bifurcation result due to Rabinowitz.

Joint work with Romildo N. de Lima (UFCG) and Alânnio B. Nóbrega (UFCG).

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Nonlocal Choquard-type equations in dimension 2

Cristina Tarsi *

We are concerned with existence and concentration of ground state solutions to nonlocal Schrödinger equations, sometimes called Choquard-type equations, in $\mathbb{R}^2$. We consider the case in which the nonlinearity exhibits exponential critical growth with respect to the Trudinger-Moser inequality. The case of a logarithmic kernel is also considered.

Joint work with C.O. Alves, D. Cassani and M. Yang.

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Recent Results on PDEs with Zero Mass

David G. Costa *

We review some results about Schrodinger Equations in $\mathbb{R}^N$ with *Zero-Mass* which were obtained in recent years regarding the topics:

- Maximum Principle
- Sub-Super Solution Method
- Existence of One-Signed and Sign-Changing Solutions

This is joint work with Siegfried Carl (Martin-Luther-Universitt Halle-Wittenberg, Germany) and Hossein Tehrani (University of Nevada Las Vegas, USA).

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On two equations modeling bridges oscillations

Ederson Moreira dos Santos *

In the first model we consider the deck of the bridge as a beam and we present sharp results about the finite space blow up of the solutions of a fourth order ODE. In the second model we see the deck of the bridge as a plate where the two short edges are hinged whereas the two long edges are free. We study a nonlocal fourth order evolution equation and we present some theorems on the stability/instability of *simple modes* motivated by a phenomenon which is visible in actual bridges and we complement these theorems with some numerical experiments.

This talk is based on joint works with Denis Bonheure (Universit libre de Bruxelles), Filippo Gazzola (Politecnico di Milano), and Vanderley Ferreira Jr (Universidade Estadual de Campinas).

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An elliptic equation with indefinite nonlinearities and exponential critical growth in $\mathbb{R}^2$

Elves A. B. Silva *

In this talk we present results on the existence, nonexistence and multiplicity of positive solutions for a class of semilinear elliptic problems involving indefinite nonlinearities with exponential critical growth of Trudinger-Moser type. The main hypothesis is that the indefinite term is the product of a weight function, having a thick zero set, and a nonlinear function with exponential critical growth satisfying a version of the Ambrosetti-Rabinowitz superlinear condition. Our proofs rely on a variational approach and sub-supersolution methods.

Joint work with Everaldo S. Medeiros (Federal University of Paraíba) and Uberlandio B. Severo (Federal University of Paraíba).

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Nonlinear higher-order eigenvalue problems: positivity of first eigenfunctions and validity of the Faber-Krahn inequality

Enea Parini *

Higher-order eigenvalue problems are known to present some additional difficulties with respect to their second-order counterparts. The lack of a maximum principle does not allow to conclude that first eigenfunctions are positive (or negative), and indeed they can be sign-changing in some cases. Similarly, standard symmetrization techniques can not be applied, so that it is not easy to identify the domain which minimizes the first eigenvalue under a volume constraint. In this talk we will present some results about the minimization of the L^1 (resp. the L^∞) norm of the Laplacian among functions with fixed L^1 (resp. L^∞) norm, which amounts to find the first eigenvalue of some nonlinear higher-order differential operators. In particular, we will present results about the positivity of first eigenfunctions, and the validity of the Faber-Krahn inequality, namely, when the domain which minimizes the first eigenvalue under a volume constraint is the ball.

Joint work with Nikos Katzourakis (Reading, UK), Bernhard Ruf and Cristina Tarsi (Milan, Italy)

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Some control and inverse problems with medical applications

Enrique Fernández-Cara *

The goal of this talk is to present some mathematical techniques that can help to understand, assess and/or solve medical problems in oncology. In particular, some recent methods in control theory and inverse problems techniques will be considered. This includes the theoretical and numerical analysis of several problems: optimal control and null controllability problems oriented to therapy in radiology; the identification of the shape of a tumor in a soft tissue and its applications to elastography, etc.

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On a Poincar type inequality and the solution of the Sternberg-Zumbrun's conjecture

Ezequiel Barbosa *

In this talk we will discuss the application of a Poincar type inequality to study sets which are stable for the volume-constrained least area problem within the Euclidean unit ball, and provide a proof for the Sternberg-Zumbrun's conjecture.

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Elliptic equations in $\mathbb{R}^2$ with exponential growth

Federica Sani *

We discuss the existence of solutions to elliptic equations of the form

$$-\Delta u + V(x)u = f(x,u) \quad \text{in} \quad \mathbb{R}^2, \quad u \in H^1(\mathbb{R}^2) \tag{0.1}$$

where the nonlinear term f has exponential growth with respect to u, and we consider different cases according to the behavior of the potential V at infinity.

When the potential V is constant – or bounded away from zero and large at infinity – the natural space for a variational study of the problem is $H^1(\mathbb{R}^2)$ – or a suitable subspace of $H^1(\mathbb{R}^2)$ – and the maximal growth which can be treated variationally is given by the classical Trudinger-Moser inequality.

When the potential V vanishes at infinity, with a prescribed decay, the problem has a variational structure in suitable (limiting) weighted Sobolev spaces where the maximal growth allowed is determined by a weighted version of the Trudinger-Moser inequality.

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Asymptotic Dirichlet and Plateau problems in Cartan-Hadamard manifolds

Ilkka Holopainen*

I will give a survey on asymptotic Dirichlet and Plateau problems in Cartan-Hadamard manifolds. My aim is to introduce three different (but related) methods to approach these problems:

(a) Perron's method and barriers at infinity;

(b) PDE-type approach based on a Caccioppoli inequality and a sub-mean-value inequality;

(c) geometric measure theoretical approach based on mass minimizing currents.

The talk is based on joint works with Jean-Baptiste Casteras (Universite Libre de Bruxelles), Jorge de Lira (UFC), Esko Heinonen (University of Helsinki), and Jaime Ripoll (UFRGS).

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Sharp regularity for the inhomogeneous porous medium equation

José Miguel Urbano *

We show that locally bounded solutions of the inhomogeneous porous medium equation

$$u_t - \operatorname{div}\left(m u^{m-1} \nabla u\right) = f \in L^{q,r}, \quad m > 1,$$

are locally Hölder continuous, with exponent

$$\gamma = \min\left\{ \frac{\alpha_0^-}{m}, \frac{[(2q-n)r-2q]}{q[(mr-(m-1)]} \right\},$$

where α_0 denotes the optimal Hölder exponent for solutions of the homogeneous case. The proof relies on an approximation lemma and geometric iteration in the appropriate intrinsic scaling.

Joint work with Damião J. Araújo (UFPB, Brazil) and Anderson F. Maia (CMUC, Portugal).

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Games for PDEs with eigenvalues of the Hessian

Julio D. Rossi *

For a function $u : \Omega \subset \mathbb{R}^N \mapsto \mathbb{R}$, we consider the Hessian, D^2u, and its ordered eigenvalues

$$\lambda_1(D^2u) \leq ... \leq \lambda_N(D^2u).$$

Here our main concern is the Dirichlet problems for the equations:

$$P_k^+(D^2u) := \sum_{i=N-k+1}^{N} \lambda_i(D^2u) = 0, \tag{0.1}$$

(note that P_k^+ is just the sum of the k largest eigenvalues)

$$P_k^-(D^2u) := \sum_{i=1}^{k} \lambda_i(D^2u) = 0, \tag{0.2}$$

(P_k^- is the sum of the k smallest eigenvalues) and, more generally, any sum of k different eigenvalues

$$P_{i_1,\ldots,i_k}(D^2u) := \sum_{i_1,\ldots,i_k} \lambda_{i_j}(D^2u) = 0. \tag{0.3}$$

These operators appear in connection with geometry but our goal is to provide a probabilistic interpretation.

We will describe games whose values approximate viscosity solutions to these equations in the same spirit as the random walk can be used to approximate harmonic functions.

Joint work with P. Blanc (U. Buenos Aires, Argentina).

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Asymptotic Symmetry for Singular Yamabe Equation

Lei Zhang *

Let (M, g) be a compact Riemannian manifold of dimension n. A celebrated theorem of Khuri-Marques-Schoen says that if $3 \leq n \leq 24$, all the solutions to the Yamabe equation are uniformly bounded, as long as the manifold is not the standard sphere. On the other hand counter-examples have been constructed for $n \geq 25$ by Brendle and Marques. In this joint work with J. Xiong we prove that for singular Yamabe equation, the solution has asymptotic symmetry near its singularity for $3 \leq n \leq 24$.

Joint work with Jingang Xiong (Beijing Normal University)

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MULTIPLE POSITIVE SOLUTIONS TO A SEMILINEAR DIRICHLET PROBLEM IN AN EXTERIOR DOMAIN

LILIANE A. MAIA*

We will present a recent work where we establish the existence of multiple positive solutions to the semilinear Dirichlet problem

$$-\Delta u + \lambda u = f(u), \qquad u \in H_0^1(\Omega_R),$$

where $\Omega_R := \{x \in \mathbb{R}^N : |u| > R\}$ with $R > 0$, $N = 2$ or $N \geq 4$, $\lambda > 0$, and the nonlinearity f is either asymptotically linear, or superlinear and subcritical at infinity. We show that the number of positive nonradial solutions becomes arbitrarily large as $R \to \infty$.

When Ω is the complement of a ball, the problem is known to have a positive radial solution; see [1] and [2]. Thus, it is natural to ask whether the solution found in [3] coincides with the radial one or not. We shall see that it does not, if R is sufficiently large. Moreover, we will show that the number of positive nonradial solutions to this problem becomes arbitrarily large, as $R \to \infty$, when $N \neq 3$.

Joint work with Mónica Clapp (Universidad Nacional Autnoma de Mxico) and Benedetta Pellacci (Universit di Napoli Parthenope).

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Weighted Trudinger-Moser inequalities and associated Liouville type equations

Marta Calanchi *

We discuss some Trudinger–Moser inequalities with weighted Sobolev norms. Suitable logarithmic weights in these norms allow an improvement in the maximal growth for integrability, when one restricts to radial functions.

The main results concern the application of these inequalities to the existence of solutions for certain mean-field equations of Liouville-type. Sharp critical thresholds are found such that for parameters below these thresholds the corresponding functionals are coercive and hence solutions are obtained as global minima of these functionals. In the critical cases the functionals are no longer coercive and solutions may not exist.

We also discuss a limiting case, in which the allowed growth is of double exponential type. Surprisingly, we are able to show that in this case a local minimum persists to exist for critical and also for slightly supercritical parameters. This allows to obtain the existence of a second (mountain-pass) solution, for almost all slightly supercritical parameters, using the Struwe monotonicity trick. This result is in contrast to the non-weighted case, where positive solutions do not exist (in star-shaped domains) in the critical and supercritical case.

Joint work with Eugenio Massa, (Universidade de São Paulo) and Bernhard Ruf (Università degli Studi di Milano).

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Existence of multiple solutions for the van der Waals-Allen-Cahn equation

Paolo Piccione *

I will discuss the existence of multiple solutions for the following nonlinear problem: for a fixed $V \in \mathbb{R}^+ =]0, +\infty[$ and $\varepsilon > 0$ small, find $u \in H_0^1(\Omega)$, and $\lambda \in \mathbb{R}$ such that

$$-\varepsilon^2 \Delta u + W'(u) = \lambda,$$

and

$$\int_\Omega u(x)\,\mathrm{d}x = V,$$

where Ω is an open bounded set in $\mathbb{R}^N$ and $W : \mathbb{R} \to \mathbb{R}$ is a function of class C^2 which satisfies the following assumptions:

(a) $W(0) = W'(0) = 0$, $W''(0) > 0$;

(b) there exists $s_0 \in]0, +\infty[$ such that

$$W(s_0) = \min\{W(s) : s \in \mathbb{R}\} < 0;$$

(c) suitable growth conditions.

The simplest example of this type of potentials is given by the non-symmetric Allen-Cahn potential:

$$W(s) = s^2(s - s_1)(s - s_2),$$

where $0 < s_1 < s_0 < s_2$. In theoretical biology equations of this type model pattern formation related to solutions which are not absolute minima of the energy. From a purely mathematical viewpoint, the above equation is also interesting due to its relation with the theory of constant mean curvature hypersurfaces.eory.

Joint work with Vieri Benci (Pisa University) and Stefano Nardulli (Federal University of Rio de Janeiro).

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Minimal time issues for Grushin type equations

Sylvain Ervedoza *

The goal of this talk is to present several sharp results on the minimal time required for observability of several Grushin-type equations. Namely, it is by now well-known that Grushin-type equations are degenerate parabolic equations for which some geometric conditions are needed to get observability properties, contrarily to the usual parabolic equations. Our results concern the Grushin operator $\partial_t - \Delta_{xx} - |x|^2\Delta_{yy}$ observed from the whole boundary in the multi-dimensional setting, from one lateral boundary in the one-dimensional setting, including some generalized version of the form $\partial_t - \partial_{xx} - (q(x))^2\partial_{yy}$ for suitable functions q ($q(x) \simeq x$ in a sense to be made precise), and the Heisenberg operator $\partial_t - \partial_{xx} - (x\partial_z + \partial_y)^2$ observed from one lateral boundary. In all these cases, our approach strongly relies on the analysis of the family of one-dimensional equations obtained by using the Fourier expansion of the equations in the y (or (y, z)) variables, and in particular the asymptotic of the cost of observability in the Fourier parameters. Combining these estimates with results on the rate of dissipation of each of these equations, we obtain observability estimates in suitably large times. We shall then explain that the times we obtain to get observability are optimal in several cases using Agmon type estimates.

Joint work with Karine Beauchard (IRMAR, Ecole Normale Supérieure de Rennes, UBL, CNRS, Campus de Ker Lann, 35170 Bruz, France) and Jérémi Dardé (Institut de Mathématiques de Toulouse ; UMR 5219 ; Université de Toulouse ; CNRS ; UPS IMT F-31062 Toulouse, France).

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35-minutes Talks

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Existence of strong traveling waves for a combustion model in a porous medium

Aparecido J. de Souza *

In this work we consider the existence of combustion fronts propagating through a one dimensional porous media consisting of oil and gas [1]. Such fronts are modeled as traveling waves of the reaction-difusion-convection PDE system

$$\begin{cases} s_t + f_x = -(hs_x)_x \\ \big((\alpha - s)\theta - \eta\epsilon s\big)_t + \big((\beta - f)\theta - \eta\epsilon f\big)_x = \big((\theta + \eta\epsilon)hs_x + \gamma\theta_x\big)_x \\ (\epsilon s)_t + (\epsilon f)_x = -(\epsilon h s_x)_x + \zeta s q \,, \end{cases}$$

where x is position, t is time, $s(x,t)$ is the gas saturation, $\theta(x,t)$ the temperature, $\epsilon(x,t)$ the fraction of burned oxygen, f is the flux function given by $f(s,\theta) = \frac{s^2}{s^2+(0.1+\theta)(1-s)^2}$, $h(s,\theta)$ is some negative function, q is the reaction rate given by

$$q(\epsilon,\theta) = \begin{cases} (1-\epsilon)A_a e^{-\frac{E}{(\theta-\theta_0)}}, \quad if \quad \theta > \theta_0, \\ 0, \quad if \quad 0 < \theta \le \theta_0 \,, \end{cases}$$

and α, β, γ, η, ζ, E, θ_0, A_a are nonnegative constants depending on the physical properties of the porous medium and fluids.

The existence of the traveling waves is reduced to the study of the existence of heteroclinic orbits connecting a hyperbolic to a nonhyperbolic equilibria of a 3×3 ODE system in which one equilibrium represents the composition of the porous medium before the passage of the combustion front and the other the composition after it. We are interested in traveling waves that are *strong* in the sense that the orbits approach the nonhyperbolic equilibrium by its stable manifold and not along its center direction. We determine two disjoint and closed intervals for θ such that each endpoint of each interval defines a strong traveling wave (combustion front) speed. The results are obtained using mainly the geometric singular perturbation theory [2] and invariant regions [3].

Joint work with J. C. da Mota (UFG).

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LOCAL EXACT CONTROLLABILITY OF A SOLIDIFICATION MODEL WITH FEW CONTROLS

Bianca M. R. Calsavara *

In this work it is analyzed a control problem with a reduced number of controls for a phase field system modeling a solidification process of materials that allow two different types of crystallization and the flow of material in the nonsolid regions. In this system we have three Allen-Cahn equations describing the phase field functions coupled to modified Navier-Stokes system and a heat equation for the temperature. It is proved that this system is locally exactly controllable to suitable homogeneous trajectories with controls acting only on the velocity field and heat equations. One of the difficulties of this work is that the three phase field equations are controlled by the velocity and temperature functions, but the coupling is multiplicative in the mentioned equations.

Joint work with Fágner D. Araruna and Enrique Fernández Cara.

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About a problem with multiple regions of singularities

Carlos Alberto Santos *

In this talk I am going to present some results about existence, uniqueness, multiplicity and regularity of solutions for the following quasilinear λ-problem involving variable exponents

$$\begin{cases} -\Delta_{p(x)}u = c(x)d(x)^{-\beta(x)}u^{-\alpha(x)} + \lambda f(x,u) \text{ in } \Omega, \\ u > 0 \text{ in } \Omega; \ u = 0 \text{ on } \partial\Omega. \end{cases}$$

One of our main interest is presenting conditions on the variable exponents, powers and $f(x,u)$ to show existence and uniqueness of $W^{1,p(x)}_{loc}(\Omega)$-solutions for situations of multiple regions of singularity in the sense that both $\beta(x)$ and $\alpha(x)$ may change signs in either Ω or $\partial\Omega$. Another interest point is establishing conditions on such terms to get multiplicity of solutions still permitting oscillations of the power $\alpha(x)$, that is, multiple regions of singularities and non-singularities of the term $u^{-\alpha(x)}$ for $u > 0$.

Joint work with Thiago Willians Ramos (Instituto Federal de Brasília - IFB) and Claudianor de Oliveira Alves (Universidade Federal de Campina Grande-UFCG).

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Observability inequalities on measurable sets for the Stokes system

Diego A. Souza *

In this talk we establish spectral inequalities on measurable sets of positive Lebesgue measure for the Stokes operator, as well as observability inequalities on space-time measurable sets of positive measure for non-stationary Stokes system. Furthermore, we provide their applications in the time optimal control problems.

Joint work with Felipe W. Chaves-Silva (Department of Mathematics, Federal University of Pernambuco) and Can Zhang (School of Mathematics and Statistics, Wuhan University).

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Up to the boundary gradient estimates in nonlinear free boundary problems

Diego Ribeiro Moreira *

In this talk, we present some ingredients that lead to gradient estimates up to the boundary in nonlinear free boundary problems. As an application, we apply these estimates to deal with singular perturbation problems of flame propagation type in combustion theory.

Joint work with Ederson M. Braga (UFC) and J. Gleison C. Da Silva (UFC-Russas).

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Interior Regularity Results for Zero-TH Order Operators Approaching the Fractional Laplacian

Disson dos Prazeres *

In this lecture we going to talk about interior regularity results for the solution $u_\epsilon \in C(\bar{\Omega})$ of the Dirichlet problem

$$\begin{cases} -\mathcal{I}_\epsilon(u) = & f_\epsilon \quad \text{in } \Omega \\ \quad u = & 0 \quad \text{in } \Omega^c. \end{cases} \quad . \tag{0.1}$$

where $-\mathcal{I}_\epsilon$ is an approximation of the well-known fractional Laplacian of order σ,as ϵ tends to zero. The purpose of this talk is to understand how the interior regularity of u_ϵ evolves as ϵ approaches zero. We going to present recent results which provide that u_ϵ has a modulus of continuity which depends on the modulus of f_ϵ, which becomes the expected Hölder profile for fractional problems, as $\epsilon \to 0$. This analysis includes the case when f_ϵ deteriorates its modulus of continuity as $\epsilon \to 0$.

Joint work with P. Felmer (CMM-UC) and E. Topp (USACH).

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Quasilinear elliptic problems using the Nehari method

Edcarlos D. da Silva *

In this talk we consider existence, multiplicity and asymptotic behavior of nonnegative solutions for a quasilinear elliptic problems driven by the Φ-Laplacian operator. One of these solutions is obtained as ground state solution by applying the well known Nehari method. The nonlinear term can be a concave-convex function which presents subcritical or critical behavior at infinity. The concentration compactness principle is used in order to recover the compactness required in variational methods.

Joint work with J. V. Goncalvez (UFG), M. L. Carvalho (UFG) and C. Goulart (UFG).

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Blow up of solutions of semilinear heat equations with almost Hénon-critical exponent

Eugenio Massa *

We study the initial value problem

$$\begin{cases} u_t - \Delta u = |x|^\alpha |u|^{\frac{4+2\alpha}{N-2}-\varepsilon} u & \text{in } B_1 \times (0,T), \\ u = 0 & \text{on } \partial B_1 \times (0,T), \\ u = u_0 & \text{in } B_1 \times \{0\}, \end{cases} \tag{0.1}$$

where B_1 is the unitary ball in $\mathbb{R}^N$, $N \geq 3$, $T \in (0, +\infty]$, $\varepsilon > 0$ is a small parameter and $\alpha > 0$ is a number which is not an even integer. We aim to proving that if $\varepsilon > 0$ is small enough, then there exists a sign changing stationary solution ψ_ε of (0.1), such that the solution of (0.1) with initial condition $u_0 = \lambda\psi_\varepsilon$ blows up in finite time for $|\lambda - 1| > 0$ small enough.

Joint work with S. Alarcón and L. Iturriaga (Universidad Técnica Federico Santa María, Chile).

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Nehari manifold and Schroödinger equation

Francisco Odair Vieira de Paiva *

We study the Schrödinger equation $-\Delta u + V(x)u = f(x,u)$ in $\mathbb{R}^N$. We assume that f is superlinear but of subcritical growth and $f(x,u)/|u|$ is nondecreasing. We also assume that V and f are periodic. We show that these equations have a ground state and that there exist infitely many solutions if f is odd in u.

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α-Navier-Stokes-Vlasov model for spray flows

Gabriela Del Valle Planas *

We investigate the interaction of a spray of particles with a Newtonian, viscous, and incompressible fluid. We consider the α-Navier-Stokes equations coupled with a Vlasov type equation to model the flow of the incompressible fluid containing small particles. We prove the global existence of weak solutions to the coupled system subject to periodic boundary conditions. The convergence of its solutions to that of the Navier-Stokes-Vlasov equations when α tends to zero is also established.

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On a Schrödinger equation in a generalized electrodynamics

Gaetano Siciliano *

We consider a system deriving from the interaction of the Schrödinger equation with a modified version of the Maxwell equation proposed by B. Podolsky in 1942. The advantage of the Maxwell lagrangian proposed by Podolsky, in contrast e.g. to the Born-Infeld Lagrangian, is that the equation involving the electromagnetic field is still linear.

More specifically the lagrangian is given by (see [1, Formula (3.9)]):

$$\begin{aligned}\mathcal{L}_{\mathrm{BP}} &= \frac{1}{8\pi}\left\{|\mathbf{E}|^2 - |\mathbf{H}|^2 + a^2\left[(\mathrm{div}\mathbf{E})^2 - \left(\nabla\times\mathbf{H} - \frac{1}{c}\dot{\mathbf{E}}\right)^2\right]\right\}\\ &= \frac{1}{8\pi}\left\{|\nabla\phi + \frac{1}{c}\partial_t\mathbf{A}|^2 - |\nabla\times\mathbf{A}|^2 + a^2\left[\left(\Delta\phi + \frac{1}{c}\mathrm{div}\partial_t\mathbf{A}\right)^2 - \left(\nabla\times\nabla\times\mathbf{A} + \frac{1}{c}\partial_t(\nabla\phi + \frac{1}{c}\partial_t\mathbf{A})\right)^2\right]\right\}\end{aligned}$$

where

$$\mathbf{E} = -\nabla\phi - \frac{1}{c}\partial_t\mathbf{A} \qquad \mathbf{H} = \nabla\times\mathbf{A}$$

is the electromagnetic field. Here $a > 0$ is a constant. Of course, whenever $a = 0$ it reduces to the Lagrangian of the classical electrodynamics.

After using the "minimal coupling rule" which describes the interaction between the Schrödinger matter field and the electromagnetic field driven by $\mathcal{L}_{\mathrm{BP}}$, the search of standing waves solutions in the purely electrostatic situation led to a system of this type in $\mathbb{R}^3$

$$\begin{cases} -\Delta u + u + q\phi u = |u|^{p-2}u \\ -\Delta\phi + \Delta^2\phi = qu^2. \end{cases}$$

where $q > 0$ has the meaning of the electric charge and all the other physical constants have been normalized. The parameter p is subcritical.

Observe that, for every fixed $u \in H^1(\mathbb{R}^3)$, the second equation has a unique solution ϕ_u in a suitable Hilbert space, however it is not homogeneous like in the classical Schrödinger-Poisson system. In particular it seems difficult to work with "Nehari methods". In spite of this, we are able, by using variational methods (in particular Mountain Pass arguments), to prove the existence of solutions, depending on p and on the value of the charge $q > 0$.

Joint work with Collaborator1 (Institution1) and Collaborator2 (Institution2).

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Boundedness of solutions of measure differential equations and dynamic equations on time scales

Jaqueline G. Mesquita *

In this talk, we investigate the boundedness results for measure differential equations. In order to obtain our results, we use the correspondence between these equations and generalized ODEs. Furthermore, we prove our results concerning boundedness of solutions for dynamic equations on time scales, using the fact that these equations represent a particular case of measure differential equations.

Joint work with M. Federson (Universidade de São Paulo), R. Grau (Universidad del Norte) and E. Toon (Universidade Federal Juiz de Fora).

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Ground states and concentration for strongly coupled elliptic systems in dimension two

Jianjun Zhang *

In this talk, we are concerned with singularly perturbed strongly coupled elliptic systems

$$\begin{cases} -\varepsilon^2\Delta\varphi + V(x)\varphi = g(\psi) & \text{in } \mathbb{R}^2, \\ -\varepsilon^2\Delta\psi + V(x)\psi = f(\varphi) & \text{in } \mathbb{R}^2, \end{cases} \tag{0.1}$$

where f, g have critical growth in the sense of Trudinger-Moser. Firstly, by using a suitable variational framework based on the generalized Nehari Manifold method, we investigate the existence of ground state solutions of the limit problem

$$\begin{cases} -\Delta u + V_0 u = g(v), \\ -\Delta v + V_0 v = f(u), \end{cases} \tag{0.2}$$

where $V_0 > 0$. We prove that actually the ground state of (0.2) does not change sign, i.e., $uv > 0$ in $\mathbb{R}^2$. Secondly, we apply this result to prove that for $\varepsilon > 0$ small, (0.1) admits a *positive* ground state solution $(\varphi_\varepsilon, \psi_\varepsilon)$ concentrating around the global minimum point of $V(x)$ as $\varepsilon \to 0$.

Joint work with Daniele Cassani (University of Insubria) and with Djairo G. De Figueiredo(UNICAMP) and João Marcos do Ó (UFPB).

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Sharp regularity estimates for fully nonlinear parabolic equations

João Vitor da Silva *

In this talk we will prove sharp regularity estimates for viscosity solutions of parabolic equations as follows

$$\frac{\partial u}{\partial t} - F(x, t, Du, D^2u) = f(x, t) \text{ in } \mathcal{Q}_1 = \mathcal{B}_1 \times (-1, 0],$$

where F is a second order fully nonlinear operator, its coefficients are merely mensurable with small enough oscillation, and $f \in L^{p,q}(\mathcal{Q}_1)$, i.e., an anisotropic Lebesgue space with exponents $p, q \in [1, \infty)$ such that $0 < \frac{n}{p} + \frac{2}{q} < 1$. Under such assumptions, we will establish local $C^{1+\zeta, \frac{1+\zeta}{2}}$ regularity estimates for such models, where the sharp value of $\zeta \in (0, 1)$ is explicitly found in terms of structural and universal parameters of the problem, i.e., ellipticity constants of the operator, dimension and integrability of the source term.

The mathematical insights for proving such a sharp $C^{1+\zeta, \frac{1+\zeta}{2}}$ regularity are based on a refined compactness method, as well as a systematic iterative approximation procedure arising from [1]. Such estimates can be found in the manuscript [2] and are an extension to obtained ones in [1] and [3].

This is joint work with Eduardo V. Teixeira (University of Central Florida - USA).

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On the spectrum of warped products and G-manifolds

José N. V. Gomes*

In this talk, we study the spectrum of warped products in order to obtain a class of G-manifolds (that contain principal bundles) which is possible to describe its generic spectrum. We establish a kind of splinting eigenvalues theorem considering a family of operators on the base of a warped product. As a consequence, we prove a density theorem for a set of warping functions that makes the spectrum of the Laplacian a warped-simple spectrum. As an application, we give an answer to the generic situation for eigenvalues of the Laplacian on a class of compact G-manifolds. In particular, we give a partial answer to a question posed by S. Zelditch [Ann. Inst. Fourier 40 (1990) 407-442] about the generic situation of multiplicity for the eigenvalues of the Laplacian on principal bundle.

Joint work with Marcus A. M. Marrocos (Federal University of ABC).

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Liouville theorems for radial solutions of semilinear elliptic equations

Leonelo Iturriaga *

Abstract

In this work we obtain some new Liouville theorems for positive, radially symmetric solutions of the equation

$$-\Delta u = f(u) \qquad \text{in } \mathbb{R}^N$$

where f is a continuous function in $[0, +\infty)$ which is positive in $(0, \infty)$. Our methods adapt to cover more general problems, where the nonlinearity is multiplied by some radially symmetric weights and/or the Laplacian is replaced by the p-Laplacian, $1 < p < N$. Some results for related elliptic systems are also obtained.

Joint work with A. Quaas (Departamento de Matemática, Universidad Técnica Federico Santa María) and J. Garca-Melin (Departamento de Análisis Matemático, Universidad de La Laguna).

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On the Bresse-Timoshenko systems

Ma To Fu *

This talk is concerned with the dynamics of the Bresse system, a recognized model for arched beams. It reduces to the classical Timoshenko system when the arch curvature ℓ vanishes. Here we discuss some recent results on the long-time dynamics of Bresse systems when ℓ tends to zero.

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Existence and multiplicity of self-similar solutions for heat equations with nonlinear boundary conditions

Marcelo F. Furtado *

We are going to talk about self-similar solutions in the half-space for linear and semilinear heat equation. Existence, multiplicity and positivity of these solutions are analyzed. Self-similar profiles are obtained as solutions of a nonlinear elliptic PDE with drift term and a nonlinear Neumman boundary condition. We consider both subcritical and critical case by employing a variational approach and deriving some compact weighted embeddings for the trace operator.

Joint works with Lucas C. Ferreira (UNICAMP), Everaldo S. Medeiros (UFPB) and Jo ?ao Pablo P. Silva (UFPA).

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Qualitative and geometric aspects of fractional elliptic equations

Olivaine Santana de Queiroz *

In the last months we have studied with several collaborators qualitative aspects of fractional elliptic equations. In particular, we study singular fractional Yamabe type equations and also some important inequalities in this fractional setting, such as Faber-Krahn and Trudinger-Moser.

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Asymptotic supnorm estimates for convection-diffusion equations and systems

Pablo Braz e Silva *

We discuss a direct method to obtain general large time bounds for the supnorms of solutions for various 1D convection-diffusion initial value problems(as, for example, in [1], [2]). We also mention some open interesting problems.

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On a class of Kirchhoff elliptic equations involving critical growth and vanishing potentials

Pedro Ubilla *

We establish the existence of positive solutions for a class of stationary Kirchhoff type equations defined in the whole $\mathbb{R}^3$ involving critical growth in the sense of the Sobolev embedding and potentials which decay to zero at infinity in some direction. In order to obtain the solution we used minimax techniques combined with an appropriated truncated argument, and a priori estimates. This results are new even for the local case which corresponds to nonlinear Schrödinger equations.

Joint work with J.M. do O(UFPB) and M.A. Souto(UFCG).

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Continuation results for retarded functional differential equations on manifolds

Pierluigi Benevieri *

We investigate the following parametrized second order retarded functional equation on a possibly noncompact manifold $M \subseteq \mathbb{R}^k$:

$$x''_\pi(t) = \lambda f(t, x_t), \quad \lambda \geq 0,$$

where: $x''_\pi(t)$ stands for the tangential part of the acceleration $x''(t) \in \mathbb{R}^k$ at the point $x(t) \in M$.

We prove existence and global continuation results for T-periodic solutions. The approach is topological and is based on the degree theory for tangent vector fields as well as on the fixed point index theory.

Joint work with Alessandro Calamai (Marche Polytechnic University), Massimo Furi (University of Florence) and Maria Patrizia Pera (University of Florence)

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A NONEXISTENCE RESULT FOR A NONHOMOGENEOUS ASYMPTOTICALLY LINEAR EQUATION

RAQUEL LEHRER*

In this talk we consider the following Schrödinger equation in $\mathbb{R}^N$, for $N \geq 3$ and $\lambda > 0$:

$$-\Delta u + \lambda u = \frac{u^3}{1 + s(x)u^2}, \tag{0.1}$$

where $s : \mathbb{R}^N \to \mathbb{R}$ is given by $s(x) = \frac{1}{1+|x|^2} + s_\infty$, with $s_\infty > 0$.

The functional associated with (0.1) is given by $I : H^1(\mathbb{R}^N) \to \mathbb{R}$,

$$I(u) = \frac{1}{2}\int_{\mathbb{R}^N} |\nabla u|^2 + \lambda u^2 dx - \int_{\mathbb{R}^N} \frac{u^2}{2s(x)} - \frac{1}{2s^2(x)} \ln(1 + s(x)u^2)dx.$$

Since $\lim_{|x|\to\infty} s(x) = s_\infty$, we can define the limit problem

$$-\Delta u + \lambda u = \frac{u^3}{1 + s_\infty u^2}, \tag{0.2}$$

and the functional

$$I_\infty(u) = \frac{1}{2}\int_{\mathbb{R}^N} |\nabla u|^2 + \lambda u^2 dx - \int_{\mathbb{R}^N} \frac{u^2}{2s_\infty} - \frac{1}{2s_\infty^2} \ln(1 + s_\infty u^2)dx$$

associated with this limit problem. It is easy to verify that the functionals I and I_∞ satisfy the unusual relation $I_\infty(u) < I(u), \forall u \in H^1(\mathbb{R}^N) \setminus \{0\}$.

Considering the Pohozaev identity related with (0.1), given by

$$\frac{N-2}{2}\int_{\mathbb{R}^N} |\nabla u|^2 dx = N\int_{\mathbb{R}^N} \frac{u^2}{2s(x)} - \frac{1}{2s^2(x)} \ln(1 + s(x)u^2) - \frac{\lambda u^2}{2} dx \tag{0.3}$$

we can define the Pohozaev manifold $\mathcal{P}$ by

$$\mathcal{P} = \left\{u \in H^1(\mathbb{R}^N) \setminus \{0\}\, ; u \quad satisfies\ (0.3)\right\}.$$

Working with projections over the Pohozaev manifold $\mathcal{P}$, and the Pohozaev manifold $\mathcal{P}_\infty$, associated with the limit problem (0.2) we can provide a characterization for the Mountain Pass level of the functional I,

$$c := \min_{\gamma\in\Gamma} \max_{0\leq t\leq 1} I(\gamma(t)).$$

where

$$\Gamma = \left\{\gamma \in C([0,1], H^1(\mathbb{R}^N)) | \gamma(0) = 0, I(\gamma(1)) < 0\right\}.$$

Theorem 1: Considering $p = \inf_{u\in\mathcal{P}} I(u)$ and c_∞, the Mountain Pass level for the functional I_∞, we have that

$$c = p = c_\infty > 0.$$

Theorem 2: The infimum $p = \inf_{u\in\mathcal{P}} I(u)$ is not a critical level for the functional I. In particular, the infimum p is not achieved.

With these results, we conclude that we need raise the energy level of the functional I in order to find a solution for equation (0.1).

Joint work with Sergio H. M. soares (USP - São Carlos), and it has the financial support of FAPESP 2016/20798-5.

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Sharp isoperimetric inequalities for small volumes in complete noncompact Riemannian manifolds of bounded geometry involving the scalar curvature

Stefano Nardulli *

We provide an isoperimetric comparison theorem for small volumes in a n-dimensional Riemannian manifold (M^n, g) with C^3 bounded geometry in a suitable sense involving the scalar curvature function. Under C^3 bounds of the geometry, if the supremum of scalar curvature function $S_g < n(n-1)k_0$ for some $k_0 \in \mathbb{R}$, then for small volumes the isoperimetric profile of (M^n, g) is less then or equal to the isoperimetric profile of the complete simply connected space form of constant sectional curvature k_0. This work generalizes Theorem 2 of [1] in which the same result was proved in the case where (M^n, g) is assumed to be compact. As a consequence of our result we give an asymptotic expansion in Puiseux series up to the second nontrivial term of the isoperimetric profile function for small volumes, generalizing our earlier asymptotic expansion [2]. Finally, as a corollary of our isoperimetric comparison result, it is shown that for small volumes the Aubin-Cartan-Hadamard's Conjecture is true in any dimension n in the special case of manifolds with C^3 bounded geometry, and $S_g < n(n-1)k_0$. Two different intrinsic proofs of the fact that an isoperimetric region of small volume is of small diameter. The first under the assumption of mild bounded geometry, i.e., positive injectivity radius and Ricci curvature bounded below. The second assuming the existence of an upper bound of the sectional curvature, positive injectivity radius, and a lower bound of the Ricci curvature.

Joint work with Luis Eduardo Osorio Acevedo (UFRJ-Universidade Federal do Rio de Janeiro).

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Short Communications

On The Extremal Parameters Curve of a Quasilinear Elliptic System of Differential Equations

Abiel Costa Macedo *

In this work we consider the following system of quasilinear elliptic equations, with indefinite super-linear nonlinearity, depending on two real parameters λ, μ:

$$\{ -\Delta_p u = \lambda |u|^{p-2}u + \alpha f |u|^{\alpha-2}|v|^{\beta} u \text{in } \Omega, -\Delta_q v = \mu |v|^{q-2} v + \beta f |u|^{\alpha}|v|^{\beta-2} v \text{in } \Omega, (u,v) \in W_0^{1,p}(\Omega) \times W_0^{1,q}(\Omega).$$

By using the Nehari manifold and the notion of extremal parameter, we extend some results concerning existence of positive solutions.

Joint work with Kaye Silva (UFG).

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Existence of multi-bump solutions for a class of elliptic problems involving the biharmonic operator

Alânnio Barbosa Nóbrega *

Using variational methods, we establish existence of multi-bump solutions for the following class of problems

$$\begin{cases} \Delta^2 u + (\lambda V(x) + 1)u = f(u), & \text{in} \quad \mathbb{R}^N, \\ u \in H^2(\mathbb{R}^N), \end{cases}$$

where $N \geq 1$, Δ^2 is the biharmonic operator, f is a continuous function with subcritical growth, $V : \mathbb{R}^N \to \mathbb{R}$ is a continuous function verifying some conditions and $\lambda > 0$ is a real constant large enough.

Joint work with Claudianor Oliveira Alves (UFCG).

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Existence and non-existence of positive solutions of quasi-linear elliptic equations involving gradient terms.

Dania González Morales *

We study the existence and non-existence of non negative solutions in the whole Euclidean space of coercive quasi-linear and fully nonlinear elliptic equations described by

$$\Delta_p u = f(u) \pm g(|\nabla u|)$$

where

$$f \in C([0,\infty)), g \in C^{0,1}([0,\infty)) \text{ are strictly increasing with } f(0) = g(0) = 0.$$

We give conditions on f and g which guarantee the existence or absence of positive solutions of this problem in $\mathbb{R}^n$. These results represent a generalization to a result obtained for the case of the Laplacian operator, by Patricio Felmer, Alexander Quaas and Boyan Sirakov.
In the particular case of the problem with plus sign on the right hand side we obtain generalized Keller- Osserman integral conditions. It turns out that different conditions are needed when $p \geq 2$ or $p \leq 2$ to deal with the existence results. The existence and non-existence in this case are established in a weak sense (the Sobolev sense).
For the problem with minus sign we show the existence also independently of the operator whenever possible to ensure the non-negativity of the non-linearity. The result of non-existence in this case is obtained independently of the gradient term.

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Nonlocal Schrödinger-Poisson systems with critical oscillatory growth

Diego Ferraz *

In this talk, we are concerned with the existence of nontrivial solutions for the following nonlinear fractional Schrödinger–Poisson system

$$\begin{cases} (-\Delta)^s u + a(x)u + K(x)\phi u = f(x,u) + g(x,u) & \text{in } \mathbb{R}^3, \\ (-\Delta)^\alpha \phi = K(x)u^2 & \text{in } \mathbb{R}^3, \end{cases}$$

in absence of compactness (unbounded domain and/or critical nonlinearities), where $0 < s < 1$, $0 < \alpha < 1$, and $2\alpha + 4s \geq 3$. The nonlinearities $f(x,t)$ and $g(x,t)$ have oscillatory subcritical and critical growth respectively, the potential $a(x)$ may change sign and the potential $K(x) \geq 0$ belongs to a suitable class of Lebesgue spaces. This class of problems involves double lack of compactness because of the unboundedness of the domain $\mathbb{R}^3$ and nonlinearities with critical log-oscillatory growth around the pure power $t \mapsto |t|^{2_s^*-2}t$ in the sense of Sobolev embedding. To overcome these difficulties we adopt an approach based in a refined version of the concentration-compactness method introduced by M. Struwe for Palais-Smale sequences for some semilinear elliptic functionals.

Joint work with João Marcos do Ó (UFPB).

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Hénon type equations with jumping nolinearities involving critical growth

Eudes Barboza *

In this work, we search for two non-trivial radially symmetric solutions of the Dirichlet problem involving a Hénon-type equation of the form

$$\begin{cases} -\Delta u &= \lambda u + |x|^\alpha k(u_+) + f(x) \quad \text{in} \quad B_1, \\ u &= 0 \qquad\qquad\qquad\qquad\quad \text{on} \quad \partial B_1, \end{cases} \tag{1}$$

where $\lambda > 0$, $\alpha \geq 0$, B_1 is a unity ball centered at the origin of $\mathbf{R}^N$ $(N \geq 3)$ and $k(s) = s^{2^*_\alpha - 1} + g(s)$ with $2^*_\alpha = 2(N+\alpha)/(N-2)$ and $g(s)$ is a C^1 function in $[0, +\infty)$ which is assumed to be in the subcritical growth range.

The proofs are based on variational methods and to ensure that the considered minimax levels lie in a suitable range, special classes of approximating functions which have disjoint support with Talenti functions (Hénon version) are constructed.

Hypotheses

Before stating our main results, we shall introduce the following assumptions on the nonlinearity g:

(g_0) $g \in C(\mathbb{R}, \mathbb{R}^+)$, $g(s) = o(s)$ when $s \to 0_+$ and $g(s) = 0$ for all $s \leq 0$.

(g_1) There exist positive constants c_1, D and s_0 and $2 < p+1 < 2^*_\alpha$ such that $g(s) \leq c_1 s^p + D$ for all $s \geq s_0$.

(g_2) There exists $c_2 > 0$ and q such that $g(s) \geq c_2 s^q$ for all $s \in \mathbb{R}^+$, where

$$\begin{cases} 2^*_\alpha - \dfrac{2N-8}{3N-8} < q+1 < 2^*_\alpha & \text{for} \quad N \geq 5; \\ (4+\alpha) - \dfrac{2}{5} < q+1 < 4+\alpha = 2^*_\alpha & \text{for} \quad N = 4; \\ (6+2\alpha) - \dfrac{2}{5} < q+1 < 6+2\alpha = 2^*_\alpha & \text{for} \quad N = 3. \end{cases} \tag{2}$$

Let us consider $\lambda_1 < \lambda_2 \leq \lambda_3 \leq \ldots \leq \lambda_j \leq \ldots$ the sequence of eigenvalues of $(-\Delta, H^1_0(B_1))$ and e_j is a j^{th} eigenfunction of $(-\Delta, H^1_0(B_1))$. Assuming (g_0) and that $\lambda \neq \lambda_j$ for all j, one can prove that ψ is a nonpositive solution of 1 if and only if it is a nonpositive solution for the linear problem

$$\begin{cases} -\Delta \psi &= \lambda \psi + f(x) \quad \text{in} \quad B_1, \\ \psi &= 0 \qquad\qquad\quad \text{on} \quad \partial B_1. \end{cases} \tag{3}$$

In order to obtain such solutions for 3, we assume that

(f_1) $f(x) = h(x) + te_1(x)$,

where $h \in L^\mu(B_1)$, $\mu > N$ and

$$\int_{B_1} he_1 \quad dx = 0. \tag{4}$$

The parameter t will be used in the proof of the first Theorem of this work.

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Statement of main results

We divide our results in two theorems. The first one deals with the first solution of the problem, which is nonpositive and is obtained by a simple remark about a linear problem related to our equation. The other theorem concerns the second solution and we need to consider the dimension which we are working. On condition (f_1), for $N \geq 5$, we only need to assume $\mu > N$ in order to recover the compactness of the functional associated to Problem 1. In dimensions $N = 4$ and 3, we should consider $\mu \geq 8$ and $\mu \geq 12$, respectively, for this purpose.

Theorem 1 (The linear problem)**.** *Assume (f_1) and $\lambda \neq \lambda_j$ for all $j \in \mathbb{N}$. Then there exists a constant $T = T(h) > 0$ such that:*

(i) *If $\lambda < \lambda_1$, there exists $\psi_t < 0$, a solution for 3 and, consequently, for 1, for all $t < -T$.*

(ii) *If $\lambda > \lambda_1$, there exists $\psi_t < 0$, a solution for 3 and, consequently, for 1, for all $t > T$.*

Furthermore, if f is radially symmetric, then ψ_t is radially symmetric as well.

Theorem 2. *Assume the existence of nonpositive radial solution ψ of 1, conditions $(g_0) - (g_2)$ and $\lambda \neq \lambda_j$ for all $j \in \mathbb{N}$. Then, 1 possesses a second radial solution provided that $f \in L^\mu(B_1)$ with $\mu \geq 12$ if $N = 3$, $\mu \geq 8$ if $N = 4$ and $\mu > N$ if $N \geq 5$.*

Joint work with Bruno Ribeiro (UFPB) and João Marcos do Ó (UFPB).

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Stationary Schrödinger equations in $\mathbb{R}^2$ with unbounded or vanishing potentials and involving concave nonlinearities

Francisco Siberio B. Albuquerque *

In this talk, we study the existence and multiplicity of solutions for the following class of stationary nonlinear Schrödinger equations:

$$-\Delta u + V(|x|)u = Q(|x|)f(u) + \lambda g(x,u), \quad x \in \mathbb{R}^2,$$

where λ is a nonnegative parameter, V and Q are unbounded or decaying radial potentials, the nonlinearity $f(s)$ may exhibit exponential growth and $g(x,s)$ is a concave term. The approach used here is based on a version of the Trudinger-Moser inequality, mountain-pass theorem and the Ekeland's variational principle in a suitable weighted Sobolev space.

Joint work with Uberlandio B. Severo (Department of Mathematics, UFPB - Universidade Federal da Paraíba).

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Multiplicity of solutions for fully nonlinear equations with quadratic growth

Gabrielle Saller Nornberg*

The study of nonlinear elliptic equations with quadratic dependence in the gradient had its beginning in the '80s, essentially with the works of Boccardo, Murat and Puel, and has been an active research object ever since. Until 2010 almost all results concerned existence of solutions in situations where uniqueness can also be obtained. Then multiplicity of bounded solutions related to nonlinear equations with quadratic growth in the gradient was observed by Sirakov, in a very particular case related to the Laplacian, for equations with constant coefficients. Further improvements were done in the last years, specially by Arcoya, de Coster, Jeanjean, Sirakov, Souplet and Tanaka, in order to give a more clear picture of the set of solutions, still for the case of the Laplacian and by using tools applicable exclusively to divergence-form second order operators.

In this talk, we will discuss some recent results obtained for non-divergence form equations, and even for fully nonlinear uniformly elliptic scenario, in the context of L^p-viscosity solutions. We also give a generalization of the Hölder regularity results of Świech-Winter to our type of equations.

Joint work with Boyan Sirakov (PUC-Rio).

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Hierarchic control for the one-dimensional wave equation in domains with moving boundary

Isaías Pereira de Jesus *

This work addresses the study of the controllability for a one-dimensional wave equation in domains with moving boundary. This equation models the motion of a string where an endpoint is fixed and the other one is moving. When the speed of the moving endpoint is less than $1 - \frac{2}{1+e^2}$, the controllability of this equation is established.

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Generalized N-Laplacian equations involving critical exponential growth and concave terms in $\mathbb{R}^N$

Jefferson Abrantes dos Santos *

In this work we establish the existence and multiplicity of nonzero and nonnegative solutions for a class of quasilinear elliptic equations, known as Generalized N-Laplacian, whose nonlinearity is allowed to enjoy the critical exponential growth with respect to a version of the Trudinger-Moser inequality and it can also contain concave terms in $\mathbb{R}^N$ $(N \geq 2)$. In order to obtain our results, we combine variational arguments in a suitable subspace of a Orlicz-Sobolev space with a version of the Trudinger-Moser inequality and Ekeland Variational Principle. In a particular case, we show that the solution is a positive ground state.

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Some results on Hamiltonian elliptic systems involving nonlinear Schrdinger equations

J. Anderson V. Cardoso *

In this talk we discuss about the following class of Hamiltonian elliptic systems involving Schrödinger equations

$$\begin{cases} -\varepsilon^2 \Delta u + V(x)u = g(x,v) & \text{in} \quad \mathbb{R}^N, \\ -\varepsilon^2 \Delta v + V(x)v = f(x,u) & \text{in} \quad \mathbb{R}^N, \end{cases} \tag{0.1}$$

where $N \geq 2$, ε is a positive parameter and $V : \mathbb{R}^N \to \mathbb{R}$ is a nonnegative, for example, locally Hölder continuous function, and $f, g : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ are continuous functions. We present results about existence, multiplicity, and non-existence of solutions for (0.1) in the subcritical, critical, and supercritical cases. Besides, we going to talk about the method of reduction by inversion for (0.1), comparing with other methods.

Joint work with do Ó, J.M. (UFPB) Ferraz, D. (UFPB), and Medeiros, E. (UFPB).

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On linearly coupled systems involving Schrödinger equations

J.C. de Albuquerque *

The study of ground state solutions for coupled systems has made great progress and attracted attention of many authors for its great physical interest. In this talk we give a survey on recent results related to the existence of ground states for several classes of linearly coupled systems involving Schrödinger equations

$$\begin{cases} Lu + V_1(x)u = f_1(x,u) + \lambda(x)v, & x \in \mathbb{R}^N, \\ Lv + V_2(x)v = f_2(x,v) + \lambda(x)u, & x \in \mathbb{R}^N, \end{cases} \tag{S}$$

where L denotes a local or nonlocal operator. We discuss the difficulties imposed by these classes of systems and the methods applied to get a ground state solution.

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Hardy type inequality and supercritical weighted Sobolev inequalities

José Francisco de Oliveira *

In this talk we give improvements to Hardy-type inequalities on weighted Sobolev spaces. Precisely, we investigate suitable conditions to

$$S_\varphi = \sup\left\{\int_0^R r^\theta |u|^{\varphi(r)}\mathrm{d}r \;|\; u\in AC_{loc}(0,R],\; u(R)=0 \text{ and } \int_0^R r^\alpha |u'|^p \mathrm{d}r = 1\right\} < +\infty$$

where $R, \alpha, \theta > 0$, $p \geq 1$ are real numbers, $\varphi(r) = p^* + r^\sigma$, with $\sigma > 0$ and p^* is the critical exponent $p^* = \frac{\theta+1}{\alpha-p+1}$, for $\alpha - p + 1 > 0$. The above supremum can be associated with an weighted Sobolev space which is a powerful tool to study a class of semilinear elliptic equations including Laplace, p-Laplace and k-Hessian operators.

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On branches of positive solutions to p-Laplacian problems at the extreme value of Nehari manifold method

Kaye Silva *

This work concerns the application of the Nehari manifold method to the study of branches of positive solutions to the problem

$$-\Delta_p u = \lambda |u|^{p-2}u + f|u|^{\gamma-2}u, \ u \in W_0^{1,p}(\Omega),$$

where Δ_p is the p-Laplacian operator, f changes signs, λ is a real parameter and $1 < p < \gamma < p^*$. A special care is given to the extreme value λ^*, which is characterized variationally by

$$\lambda^* = \inf \left\{ \frac{\int |\nabla u|^p}{\int |u|^p}, \ u \in W_0^{1,p}(\Omega), \ \int f|u|^\gamma \geq 0 \right\}.$$

The main result deals with the existence of two positive solutions when $\lambda \in (\lambda_1, \lambda^* + \varepsilon)$.

Joint work with Yavdat Il'yasov (UFA-Russia).

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Regularity up to the Boundary for solutions of Fully Nonlinear Ellipt Equations

Marcelo Drio dos Santos Amaral *

Abstract

We provide regularity results at the boundary for viscosity solutions to second order fully nonlinear uniformly elliptic equations in the form $F(D^2u(x), Du(x), x) = f(x)$ in Ω even in the case when the source function lies in the borderline cases of the theory.

1 Introduction

We know that it is possible to obtain regularity up to the boundary, putting toguether interior regularaity and boundary regularity if the last ones are available. So, thanks to the recent developments obtained by Teixeira [1] for fully nonlinear second-order uniformly elliptic equations of the form

$$F(D^2u(x), x) = f(x) \quad \text{em} \quad \Omega, \tag{RI}$$

where it was obtained interior regularity for viscosity solutions, the appropriate weak notion of solutions for (RI) and boundary regularity estimates available in the literature, see for example [2], [3], [4] and [5], just to cite a few, we can obtain the following glogal regularity results. summarized in the following table

2 Main Results

We have to extend the interior estimates obtained by Teixeira for solutions to gradient dependent equations of the form

$$F(D^2u(x), Du(x), x) = f(x) \quad \text{em} \quad \Omega. \tag{RI}$$

So, this is our fist difficulty. In fact, we are going to flatten the domain Ω by a change of variables to $B_1^+ := \{x \in B_1 : x_n > 0\}$ and gradient terms are unavoidable for the change of variables. So, we have to suppose Ω a smooth domain (C^2 is sufficient).

We are going to present the following regularity estimates for solutions of

$$\begin{cases} F(D^2u(x), Du(x), x) & = & f(x) & \text{em} & \Omega \\ u & = & \varphi & \text{sobre} & \partial\Omega \end{cases} \tag{GR}$$

summarized in the following table: where $alpha_0$ is the universal optimal exponent for solutions of the respective

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homogeneous, constant coefficient case (the *tangential problem*) and ε is the escauriaza constant (see [Es93]). It is worth nothing that in the case $f \in BMO$ we need $C^{2,\varepsilon}$ a priori estimates for the Tangential Problem (the respective, homogeneous contant coefficient case), since $C^{1,Log-Lip}$ is more regular than the Tangential Problem that is merely $C^{1,\alpha}$.

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Existence of ground states for a quasilinear coupled system in $\mathbb{R}^N$

Maxwell L. Silva *

In this work we consider the following class of quasilinear coupled systems

$$(S_\theta)\begin{cases} -\Delta u + a(x)u - \Delta(u^2)u = g(u) + \alpha\theta\lambda(x)|u|^{\alpha-2}u|v|^\beta, & x \in \mathbb{R}^N, \\ -\Delta v + b(x)v - \Delta(v^2)v = h(v) + \beta\theta\lambda(x)|v|^{\beta-2}v|u|^\alpha, & x \in \mathbb{R}^N, \end{cases}$$

where $N \geq 3$ and $a, b : \mathbb{R}^N \to \mathbb{R}$ are positive potentials, $\lambda : \mathbb{R}^N \to \mathbb{R}$ is a suitable continuous function, $\theta > 0$ and $\alpha, \beta > 2$ satisfying $\alpha + \beta < 2.2^*$. On the nonlinear terms we assume that g, h are in C^1 class and are superlinear functions at infinity and at the origin. The main theorem is stated without the well known Ambrosetti-Rabinowitz condition at infinity. Using a change of variable, we turn the quasilinear coupled system into a nonlinear coupled system, where we establish a variational approach based on Nehari method.

Joint work with Edcarlos D. Silva (UFG) and J.C. de Albuquerque (UFG).

1 Introduction

We look for ground states for the general class of quasilinear coupled systems involving Schrödinger equations (S_θ). This class of systems imposes some difficulties. The first one is that the energy functional associated to System (S_θ) is not well defined in the whole space $H^1(\mathbb{R}^N)^2$. Thus, motivated by seminal works [1, 2, 3, 4, 5, 6] we also use a change of variable to reformulate our initial problem, obtaining a nonlinear coupled system. After change of variable, the modified problem has an associated energy functional well defined in the whole space $H^1(\mathbb{R}^N)^2$ and the solutions are related with solutions of the initial System (S_θ). The second difficulty is the lack of compactness due to the fact that the system is defined in the whole Euclidean space $\mathbb{R}^N$. Moreover, System (S_θ) involve strongly coupled Schrödinger equations because of the coupling terms in the right hand side. We emphasize that we do not use the well known Ambrosetti-Rabinowitz condition. We suppose that the potentials a, b satisfy the following hypotheses:
(a_o) $a, b, \lambda \in C(\mathbb{R}^N, \mathbb{R})$ are 1-periodic functions;
(a_1) $a(x) \geq a_0$ and $b(x) \geq b_0$ for some $a_0, b_0 > 0$;
(a_2) $\lambda(x) \geq 0$ for all $x \in \mathbb{R}^N$ and $\lambda(x) > 0$ for all $x \in \Omega$, for some $\Omega \subset \mathbb{R}^N$ such that $|\Omega| < +\infty$;
(g_o) $g, h \in C^1(\mathbb{R}, \mathbb{R})$;
(g_1) $|g(t)| \leq C\left(1 + |t|^{p-1}\right)$, $\quad |h(t)| \leq C\left(1 + |t|^{p-1}\right)$, $\quad$ for all $t \in \mathbb{R}$ for some $C > 0$ and $p \in (4, 2 \cdot 2^*)$;
$(g_2)$$\lim_{t\to 0} \dfrac{g(t)}{t} = 0, \quad \lim_{t\to 0} \dfrac{h(t)}{t} = 0$;
(g_3) $\lim_{|t|\to+\infty} \dfrac{g(t)}{t^3} = +\infty, \quad \lim_{|t|\to+\infty} \dfrac{h(t)}{t^3} = +\infty$;
(g_4)The functions $t \to \dfrac{g(t)}{t^3}, \quad t \to \dfrac{h(t)}{t^3}$ are strictly increasing in $|t|$;
(g_5) There holds $0 \leq G(t) \leq G(|t|)$ and $0 \leq H(t) \leq H(|t|)$, for all $t \in \mathbb{R}$.

Theorem *Under the above hipothesis, there exists $\theta_0 > 0$ such that the System (S_θ) has at least one positive ground state solution, for all $\theta \geq \theta_0$.*

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Minimal hypersurfaces and the Allen-Cahn equation on closed manifolds

Pedro Gaspar *

Since the late 70s parallels between the theory of phase transitions and critical points of the area functional have helped us to understand variational properties of solutions to semilinear elliptic PDEs of the form

$$-\varepsilon\Delta u + W'(u)/\varepsilon = 0 \tag{1}$$

for $u : M \to \mathbb{R}$ defined in a Riemannian manifold M, where W is a double-well potential, and spaces of hypersurfaces which minimize the area in an appropriate sense. We will discuss some recent developments in this direction which extend well-known analogies regarding minimizers of the associated energy functional to more general variational solutions.

Borrowing ideas from the min-max theory of minimal hypersurfaces, we construct variational solutions of equation (1) – known as the *Allen-Cahn equation* – in a closed manifold, and study the asymptotic growth of the corresponding critical values as well as solutions with least non-trivial energy. This is joint work with M.A.M. Guaraco.

Furthermore we obtain an upper bound for the stability index of the minimal hypersurfaces which arise from solutions with uniformly bounded energy and index as $\varepsilon \downarrow 0$ in terms of the Morse index of these solutions by comparing the second inner variation of the energy functional to the second variation of the area.

Joint work with Collaborator1 (Institution1) and Collaborator2 (Institution2).

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Asymptotic Behaviour of solutions for a coupled elliptic system in the punctured ball

Rayssa Caju *

Our main goal is to study the asymptotic behavior near an isolated singularity of local solutions for strongly coupled critical elliptic systems of the form

$$-\Delta_g u_i + \sum_{j=1}^{2} A_{ij}(x) u_j = \frac{n(n-2)}{4} |\mathcal{U}|^{\frac{4}{n-2}} u_i \tag{0.1}$$

which are defined in the punctured unit ball, where g a smooth Riemannian metric on $B_1^n(0)$ and A is a C^1 map from the unit ball to the vector space of symmetrical 2×2 real matrices.

Since from the viewpoint of conformal geometry our systems are pure extensions of Yamabe-type equations in the strongly coupled regime, there has been considerable interest in recent years in proving compactness results for this type of systems. Such type of problems provides a natural background for the interplay between geometry and asymptotic analysis.

We prove a sharp result on the removability of the isolated singularity for all components of the solutions when the dimension is less than or equal to five and minus the potential A of the operator is cooperative.

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Symmetry results for positive solutions of fully nonlinear nonlocal operators

Ricardo Costa *

In this work, we study symmetry property of positive solutions for Fully Nonlinear integro-differential equations

$$\begin{cases} \mathfrak{M}^-(u) = f(u) & \text{in } B_1(0) \\ u = 0 & \text{in } \mathbb{R}^N \setminus B_1(0), \end{cases}$$

where $N \geq 1$, $x \in B_1(0) = \{x \in \mathbb{R}^N : |x| < 1\}$, the operator $\mathfrak{M}^-$ is a nonlocal extremal Pucci operator type. We would like of the establish the nonlocal counterpart of the result of daLio and Sirakov in [2], as well as [3] is a nonlocal counterpart of the [4]. The extremal Pucci operator considered here are motivated by paper [1] due to Caffarelli and Silvestre, and are defined as follow

$$\mathfrak{M}^-_{\mathcal{S}}(u) = \ inf_{L\in\mathcal{S}} L(u)$$

where $\mathcal{S}$ is a class of integro-differential operators of the form

$$L(u)(x) = \ int_{\mathbb{R}^N}(u(y) - u(x))K(|x-y|)dy,$$

satisfying

(i) the potential $K(y) = K(|y|)$ is radially symmetric and decreasing

(ii) there exists $0 < \mathbf{c} \leq \mathbf{C}$ such that

$$\frac{\mathbf{c}}{|y|^{N+2s}} \leq K(y) \leq \frac{\mathbf{C}}{|y|^{N+2s}}, \tag{0.1}$$

with $0 < s < 1$ and

(iii) the integral

$$\int_{R^N} \frac{|y|^2}{1+|y|^2} K(y)dy \qquad \text{is finite.}$$

In our proof we make use a Maximum Principle for small domain to start the moving planes to obtain the symmetry results of positive solutions.

Joint work with Disson Soares dos Prazeres (Federal University of Sergipe).

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Positivity properties of a higher order parabolic equation

Vanderley Ferreira Junior *

We discuss weak positivity properties of solutions of the initial value problem in R^n for the homogeneous parabolic equation of real order $\alpha > 0$,

$$\begin{cases} u_t + (-\Delta)^\alpha u = 0, \\ u(x,0) = u_0(x). \end{cases} \tag{0.1}$$

Our results regard eventual local positivity of solutions with nonnegative, compactly supported initial data.

Joint work with Lucas Catão de Freitas Ferreira (IMECC – Unicamp).

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COEXISTENCE STATES IN A CROSS-DIFFUSION SYSTEM OF A PREDATOR-PREY MODEL

Willian Cintra *

In this work we study the existence and non-existence of coexistence states of the following Lotka-Volterra predator-prey system with cross-diffusion

$$\begin{cases} -\nabla\left[\dfrac{1}{R(v)}\nabla u - \dfrac{uR'(v)\nabla v}{R(v)[R(v)+g(v)]}\right] = u(\lambda - u + bv) & \text{in } \Omega, \\ -d_v\Delta v = v(\mu - v - cu) & \text{in } \Omega, \\ u = v = 0 & \text{on } \partial\Omega. \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$, $N \geq 1$, is a bounded domain with a smooth boundary, $d_v, b, c > 0$ are positive constants, $R, g : [0, \infty) \to \mathbb{R}$ are functions of class $\mathcal{C}^{2,\gamma}$ and $\mathcal{C}^{1,\gamma}$, $0 < \gamma < 1$, respectively, such that

$$R(0) > 0 \quad \text{and} \quad g(s),\ R'(s) > 0,\ C \geq R(s)\ \forall s \in [0, \infty),$$

for some positive constant C. The above system was proposed in [1] and it is a particular version of the original model proposed in [2]. From the ecological point of view the functions u and v denote the population densities of the predator and prey, respectively, the left sides of the equations for u and v are dispersal (or diffusion) terms: the first term contains the dispersal term of predator and the second one incorporates the dispersal term of prey, they describe the spatial movement of species. The right sides are the classical predator-prey Lotka-Volterra reaction functions. The term $uR'(v)/(R(v)[R(v) + g(v)]$ is called cross-diffusion term, where R describes the turning rate of predator and g describes the predator satiation. This is the main contribution of [2] in order to obtain a more realistic model to describe the behaviour of predator-prey.

We use mainly bifurcation methods and a priori bounds. Actually, we extend the bifurcation result of [3] (see also [4]) for semilinear systems to the quasilinear case (1). Thus, we give conditions on the parameters λ and μ to ensure existence and non-existence of coexistence states. We also compare our results with the classical linear diffusion predator-prey model. Our results suggest that when there is no abundance of prey, the predator needs to be a good hunter to survive.

Joint work with Crístian Morales-Rodrigues (Univ. de Sevilla, Sevilla).

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Poster Sessions

Lions' maximal regularity problem

Achache Mahdi *

We report on recent progress on maximal Lp-regularity for evolution equations with time-dependent operators. These operators are associated with time dependent sesquilinear forms $\mathfrak{a}(t)$ on a Hilbert space. J. L. Lions (1960) proved the first results on maximal $L2$-regularity provided the forms $\mathfrak{a}(t)$ are C^1 (with respect to t). He then asked the problem whether this C^1 assumption is necessary. This problem was solved only recently. We discuss recent results on this problem and give some applications.

Joint work with El Maati Ouhabaz (Université de Bordeaux).

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SOME CLASS OF SCHRÖDINGER EQUATIONS INVOLVING LAPLACIAN AND P-LAPLACIAN OPERATORS WITH VANISHING POTENTIALS

Cláudia Santana *

Abstract

In this paper we study the existence of weak positive solutions for the following class of quasilinear Schrödinger equations

$$-\Delta_p u - \Delta u + V(x)u = f(u) \quad \text{in} \quad \mathbb{R}^N,$$

where f satisfies some "mountain–pass" type assumptions and V is a nonnegative continuous function. We give a special attention to the case when V may eventually vanish at infinity. Our arguments are based on penalization techniques, variational methods and Moser iteration scheme.

Key words. Quasilinear problem, Variational Methods, Mountain–pass theorem, Standing wave solution

Joint work with João Marcos do Ó (UFPB) and Elisandra Gloss (UFPB).

1 Introduction

This paper concerns the existence of weak solutions of the following nonlinear field equation

$$\begin{cases} -\Delta_p u - \Delta u + V(x)u = f(u) & \text{in} \quad \mathbb{R}^N, \\ u > 0 & \text{in} \quad \mathbb{R}^N, \\ u \in \; D_r^{1,2}(\mathbb{R}^N) \cap D_r^{1,p}(\mathbb{R}^N), \end{cases} \tag{P}$$

where $2 < p < N$, $\Delta_p u := \text{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplace operator and $p^* = Np/(N-p)$ is the critical Sobolev exponent. We assume that the nonlinearity $f : \mathbb{R} \to \mathbb{R}$ is a continuous function satisfying:

(f_1) There exists $\gamma \in (1, p^*-1]$ such that $\lim\limits_{s\to 0^+} \dfrac{f(s)}{s^\gamma} = 0$;

(f_2) $\lim\limits_{s\to +\infty} \dfrac{f(s)}{s^{p^*-1}} = 0$;

(f_3) (Ambrosetti-Rabinowitz condition) there exists $p < \theta \leq p^*$ such that

$$0 < \theta F(s) \leq sf(s) \quad \text{for all } s > 0, \quad \text{where} \quad F(s) \equiv \int_0^s f(t)dt,$$

and the potential $V : \mathbb{R}^N \to \mathbb{R}$ satisfies the following conditions:

(V_1) $\liminf_{|x|\to\infty} |x|^\alpha V(x) > 0$, where $\alpha = (N-2)(\gamma-1)/2$;

(V_2) $V \in C(\mathbb{R}^N, [0, \infty))$ is a radial function.

The main result of this paper is presented below:

Theorem 1.1. *Assume* $(f_1) - (f_3)$ *and* $(V_1) - (V_2)$. *Then there exists a positive solution for Problem* (P).

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On quasilinear elliptic equations with singular nonlinearity

Esteban da Silva *

We deal with the study of radial solutions of some quasilinear differential equations on model manifolds with singular nonlinearity and Dirichlet boundary condition. The mathematical interest lies on the loss of variational properties due to this singularity. Moreover, even the topological methods carry some limitations. Recently, this equations has received much attention due to its applicability on the modeling of micro devices.

Joint work with J. M. do Ó (Federal University of Paraíba).

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Quasilinear Schrdinger equations with unbounded or decaying potentials

Gilson Mamede de Carvalho *

In this work we are concerned with the existence of solution for *quasilinear Schrödinger equations* of the form

$$i\frac{\partial\psi}{\partial t} = -\Delta\psi + W(x)\psi - \eta(x,|\psi|^2)\psi - \kappa\left[\Delta\rho(|\psi|^2)\right]\rho'(|\psi|^2)\psi, \tag{0.1}$$

where $\psi : \mathbb{R}^N \times \mathbb{R} \to \mathbb{C}$, κ is a real constant, $W : \mathbb{R}^N \to \mathbb{R}$ is a given potential and $\eta : \mathbb{R}^N \times \mathbb{R}_+ \to \mathbb{R}$, $\rho : \mathbb{R}_+ \to \mathbb{R}$ are suitable functions. Quasilinear equations of the form (0.1) appear naturally in mathematical physics and have been derived as models of several physical phenomena corresponding to various types of nonlinear terms ρ. Here, we consider the case where $\rho(s) = s$, $\kappa > 0$ and our main interest is in the existence of *standing wave solutions*, that is, solutions of type

$$\psi(x,t) = \exp(-i\mathcal{E}t)u(x),$$

where $\mathcal{E} \in \mathbb{R}$ and $u \geq 0$ is a real function. A simple computation shows that ψ satisfies (0.1) if and only if the function $u(x)$ solves the quasilinear equation

$$-\Delta u + V(x)u - \kappa[\Delta(u^2)]u = g(x,u), \quad x \in \mathbb{R}^N, \tag{0.2}$$

where $V(x) := W(x) - \mathcal{E}$ is the new potential and $g(x,u) := \eta(x,u^2)u$ is the new nonlinear term.

The equation (0.2) has attracted a lot of attention of many researchers and some existence and multiplicity results have been obtained. We want to deal with equation (0.2) where the potential V verifies the conditions:

i) $\limsup_{|x|\to 0} V(x) = +\infty$ (singular at origin);

ii) $\liminf_{|x|\to\infty} V(x) = 0$ (vanishing at infinity).

More precisely, we are concerned with problems of the form

$$\begin{cases} -\Delta u + V(|x|)u - \kappa[\Delta(u^2)]u = Q(|x|)g(u), & x \in \mathbb{R}^N, \\ u(x) \to 0 \quad \text{as} \quad |x| \to \infty. \end{cases} \tag{P}$$

The main purpose is to show that, using a variational framework based on a suitable weighted Orlicz space, it is possible to find sufficient conditions for existence of nonnegative and nonzero solutions for (P), where $N \geq 3$, $V, Q : (0,\infty) \to \mathbb{R}$ are continuous and satisfy convenient assumptions at the origin and infinity, and the nonlinear term $g : \mathbb{R} \to \mathbb{R}$ is continuous and has adequate growth conditions for this class of problems.

More precisely we make the following assumptions on the potentials V and Q:

(V_1) $V : (0,\infty) \to \mathbb{R}$ is continuous, $V(r) \geq 0$ for all $r > 0$ and there exist $a \in \mathbb{R}$ and $a_0 \geq -2$ such that

$$0 < \liminf_{r\to 0^+} \frac{V(r)}{r^{a_0}} \leq \limsup_{r\to 0^+} \frac{V(r)}{r^{a_0}} < \infty \quad \text{and} \quad 0 < \liminf_{r\to +\infty} \frac{V(r)}{r^{a}};$$

(Q_1) $Q : (0,\infty) \to \mathbb{R}$ is continuous, $Q(r) > 0$ for all $r > 0$ and there exist $b, b_0 \in \mathbb{R}$ such that

$$0 < \liminf_{r\to 0^+} \frac{Q(r)}{r^{b_0}} \leq \limsup_{r\to 0^+} \frac{Q(r)}{r^{b_0}} < \infty \quad \text{and} \quad \limsup_{r\to +\infty} \frac{Q(r)}{r^{b}} < \infty,$$

where

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(i) $b_0 > b \quad \text{if} \quad b \geq -2 \geq a$;

(ii) $b_0 \geq b$ and $b_0 > -2 \quad \text{if} \quad b \leq \max\{a, -2\}$.

By (V_1) and (Q_1) the potentials V and Q can be singular at the origin and vanishing at infinity, as well as can be unbounded at infinity.

In order to establish the hypotheses on the nonlinearity $g(s)$, we introduce the following numbers:

$$\alpha := \begin{cases} \dfrac{2(N+b)}{N-2}, & \text{if } b \geq -2 \text{ and } a \leq -2; \\ 2, & \text{if } b \leq \max\{a, -2\}; \end{cases}$$

and

$$\beta := \frac{2(N+b_0)}{N-2}.$$

The numbers α and β are related with some embedding results, which are important for us to use the variational methods. Throughout this work the following hypotheses on $g(s)$ will be assumed:

(g_1) $g : \mathbb{R} \to \mathbb{R}$ is continuous and $g(s) = o(|s|^{\alpha-1})$ as $s \to 0$;

(g_2) there exist $p \in (\alpha, 2\beta)$ and $C_1 > 0$ such that

$$|g(s)| \leq C_1(1 + |s|^{p-1}), \ \forall \ s \in \mathbb{R};$$

(g_3) there exists $\mu > 2$ such that

$$0 < 2\mu G(s) := 2\mu \int_0^s g(t)dt \leq sg(s), \ \forall \ s > 0.$$

Under these conditions, we have our main result.

Teorema 0.1. *Suppose that (V_1), (Q_1), $(g_1) - (g_3)$ are satisfied with $b \geq -\frac{N+2}{N}$ in $(Q_1)_{(i)}$, $b \neq -2$ in $(Q_1)_{(ii)}$, $a_0 \geq b_0$ and $p > \max\{4, \alpha + 1\}$. Then, problem (P) has a nonnegative and nonzero solution $u \in X_{rad}$.*

Where

$$X_{rad} := \left\{ v \in D^{1,2}_{rad}(\mathbb{R}^N) : \int_{\mathbb{R}^N} V(|x|)|v|^2 \, dx < \infty \right\}$$

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Sharp Regularity For The Degenerate Doubly Nonlinear Parabolic Equation

janielly Gonçalves Araújo *

The aim of this paper is to obtain sharp regularity estimates for locally bounded solutions of the degenerate doubly nonlinear equation

$$u_t - \operatorname{div}(mu^{m-1}|\nabla u|^{p-2}\nabla u) = f,$$

where $m > 1$, $p > 2$ and $f \in L^{q,r}$. More precisely, we show that solutions are locally of class $C^{0,\beta}$, where β depends explicitly only on the optimal Hölder exponent for solutions of the homogeneous case, the integrability of f, the constants p, m and the space dimension n.

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Hierarchic control for the wave equation

Luciano Cipriano da Silva *

This paper deals with the hierarchical control of the wave PDE. We use Stackelberg-Nash strategies. As usual, we consider one leader and two followers. To each leader we associate a Nash equilibrium corresponding to a bi-objective optimal control problem; then, we look for a leader that solves an exact controllability problem. We consider linear and semilinear equations.

Joint work with Fágner Dias Araruna (Universidade Federal da Paraíba - UFPB) and Enrique Fernández-Cara (Universidad de Sevilla - US).

Statement of the problem

Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with boundary Γ of class C^2 and let us assume that $T > 0$. Let us consider the small open nonempty sets $\mathcal{O},\ \mathcal{O}_1,\ \mathcal{O}_2 \subset \Omega$. We consider the cylinder $Q = \Omega \times (0,T)$ with lateral boundary $\Sigma = \Gamma \times (0,T)$. By $\nu(x)$ we denote the outward unit normal to Ω at the point $x \in \Gamma$.

Let us consider the following system

$$\begin{cases} y_{tt} - \Delta y + a(x,t)y = F(y) + f1_{\mathcal{O}} + v^1 1_{\mathcal{O}_1} + v^2 1_{\mathcal{O}_2} & \text{in} \quad Q, \\ y = 0 & \text{on} \quad \Sigma, \\ y(\cdot\,,0) = y^0,\ y_t(\cdot\,,0) = y^1 & \text{in} \quad \Omega, \end{cases} \tag{0.1}$$

where $a \in L^\infty(Q)$, $f \in L^2(\mathcal{O} \times (0,T))$, $v^i \in L^2(\mathcal{O}_i \times (0,T))$ $(i = 1,2)$, $F : \mathbb{R} \to \mathbb{R}$ is a locally Lipschitz-continuous function, $(y^0, y^1) \in H_0^1(\Omega) \times L^2(\Omega)$ and the notation 1_A indicates the characteristic function of A.

The main goal of this article is to analyze the hierarchic control of (0.1) and, in particular, to prove that the Stackelberg–Nash strategy allows to solve the exact controllability problem.

Main results

Let $x_0 \in \mathbb{R}^n \setminus \overline{\Omega}$ be given and let us consider the following set

$$\Gamma_+ := \{x \in \Gamma;\ (x - x_0)\cdot \nu(x) > 0\}$$

and the function $d : \overline{\Omega} \to \mathbb{R}$, with $d(x) = |x - x_0|^2$ for all $x \in \overline{\Omega}$. We also define

$$R_0 := \min\{\sqrt{d(x)} : x \in \overline{\Omega}\} \quad \text{and} \quad R_1 := \max\{\sqrt{d(x)} : x \in \overline{\Omega}\}. \tag{0.2}$$

We will impose the following assumption:

$$\exists \delta > 0 \ \text{ such that } \ \mathcal{O} \supset \mathcal{O}_\delta(\Gamma_+) \cap \Omega, \tag{0.3}$$

where

$$\mathcal{O}_\delta(\Gamma_+) = \{x \in \mathbb{R}^n;\ |x - x'| < \delta,\ x' \in \Gamma_+\}\,.$$

A trajectory to (0.1) is a solution to the system

$$\begin{cases} \bar{y}_{tt} - \Delta \bar{y} + a(x,t)\bar{y} = F(\bar{y}) & \text{in} \quad Q, \\ \bar{y} = 0 & \text{on} \quad \Sigma, \\ \bar{y}(\cdot\,,0) = \bar{y}^0,\ y_t(\cdot\,,0) = \bar{y}^1 & \text{in} \quad \Omega. \end{cases} \tag{0.4}$$

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In the linear case ($F \equiv 0$), we have the following result on the exact controllability of (0.1):

Teorema 0.1. *Suppose that $T > 2R_1$, $F \equiv 0$ and the constants $\mu_i > 0$ $(i = 1, 2)$ are large enough, depending on Ω, $\mathcal{O}$, the $\mathcal{O}_i$, the $\mathcal{O}_{i,d}$, T and $\|a\|_{L^\infty(Q)}$. Then, for any data $(y^0, y^1) \in H_0^1(\Omega) \times L^2(\Omega)$, there exist a control $f \in L^2(\mathcal{O} \times (0,T))$ and an associated Nash equilibrium pair $(v^1, v^2) = (v^1(f), v^2(f))$ such that the corresponding solution to* (0.1) *satisfies $y(\cdot, T) = \bar{y}(\cdot, T)$.*

The following results hold in the semilinear case:

Teorema 0.2. *Assume that $T > 2R_1$, $F \in W^{1,\infty}(\mathbb{R})$ and the $\mu_i > 0$ $(i = 1, 2)$ are sufficiently large, depending on Ω, $\mathcal{O}$, the $\mathcal{O}_i$, the $\mathcal{O}_{i,d}$, T and $\|F\|_{W^{1,\infty}}$. Then, for any data $(y^0, y^1) \in H_0^1(\Omega) \times L^2(\Omega)$, there exist a control $f \in L^2(\mathcal{O} \times (0,T))$ and an associated Nash quasi-equilibrium pair $(v^1, v^2) = (v^1(f), v^2(f))$ such that the corresponding solution to* (0.1) *satisfies $y(\cdot, T) = \bar{y}(\cdot, T)$.*

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Small volumes implies small diameters, via an intrinsic monotonicity formula in Riemannian manifolds

Luis Eduardo Osorio Acevedo *

We want present another purely intrinsic proof that for small volumes isoperimetric regions are of small diameter in manifolds with some type of bounded geometry based on a monotonicity formula for varifolds of bounded generalized mean curvature which allows us to use an argument inspired from the correspondent extrinsic proof of [1] and combining it with our cut and paste argument to give finally the principal result of this poster. The monotonicity formula that we use here is an adaptation of Theorem 2.1 and Proposition 2.2 of [2] to our intrinsic Riemannian context via Hessian comparison theorems for the distance function. At our knowledge this is the first time that such an intrinsic approach appears in the literature, although being a very natural one. The applications of this methods are wide and opens the doors for extending in a rigorous way to a Riemannian ambient manifold the geometric measure theory known in $\mathbb{R}^n$, without using the Nash's isometric embedding theorem.

Joint work with Stefano Nardulli (UFRJ - IM).

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Study of an anisotropic nonlinear elliptic equation

Rajae Bentahar *

The objective is the study of an elliptic-type nonlinear anisotropic problem involving a Leray-Lions type operator with irregular data. We study solutions in the Lebesgue space

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On Lane-Emden systems with singular nonlinearities and applications to MEMS

Rodrigo Clemente *

We analyse the Lane-Emden system

$$\begin{cases} -\Delta u = \dfrac{\lambda f(x)}{(1-v)^2} & \text{in} \quad \Omega \\ -\Delta v = \dfrac{\mu g(x)}{(1-u)^2} & \text{in} \quad \Omega \\ 0 \leq u, v < 1 & \text{in} \quad \Omega \\ u = v = 0 & \text{on} \quad \partial\Omega \end{cases} \tag{$S_{\lambda,\mu}$}$$

where λ and μ are positive parameters and Ω is a smooth bounded domain of $\mathbb{R}^N$ $(N \geq 1)$. Here we prove the existence of a critical curve Γ which splits the positive quadrant of the (λ, μ)-plane into two disjoint sets $\mathcal{O}_1$ and $\mathcal{O}_2$ such that the problem $(S_{\lambda,\mu})$ has a smooth minimal stable solution (u_λ, v_μ) in $\mathcal{O}_1$, while for $(\lambda, \mu) \in \mathcal{O}_2$ there are no solutions of any kind. We also establish upper and lower estimates for the critical curve Γ and regularity results on this curve if $N \leq 7$. Our proof is based on a delicate combination involving maximum principle and L^p estimates for semi-stable solutions of $(S_{\lambda,\mu})$.

Joint work with João Marcos Bezerra do Ó (Universidade de Brasília).

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Pitchfork Bifurcation for the Nonlocal Evolution Equation

Rosangela Teixeira Guedes *

Let us consider the nonlocal evolution equation

$$\frac{\partial u(t,x)}{\partial t} = -u(t,x) + g(\lambda(J*u)(t,x)) \tag{0.1}$$

λ is a positive constant, J a function of class $C^1(\Omega)$, $\Omega \subset \Re^N$ open and limited, whose integral of J over $\Re^N$ is equal to one, $N \leq 1$, u is a real function in $]0,\infty[\times\Re^N$. Let the operator $F :]0,\infty[\times L^2(\Omega) \longrightarrow L^2(\Omega)$ defined by

$$F(\lambda, u) = -u + g(\lambda J * u)$$

where

$$(J*u)(x) = \int_\Omega J(x-y)u(y)dy,$$

and the function $g : \Re \longrightarrow \Re$. The assumptions that the eigenvalues of operator T are simple, also $g(0) = g^{''}(0) = 0, g'(0) > 0, g^{'''}(0) < 0$, we prove that the nonlocal evolution equation(1) has stable null equilibrium and from the equilibrium null pitchfork bifurcation.

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Joint work with Antônio Luiz Pereira (Matemática Aplicada-IME/USP-Orientador).

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EXISTENCE OF BOUND AND GROUND STATES FOR A CLASS OF KIRCHHOFF-SCHRÖDINGER EQUATIONS INVOLVING CRITICAL TRUDINGER-MOSER GROWTH

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Abstract

In this work we discuss the existence of bound and ground state solutions for a class of fractional Kirchhoff equations defined on the whole real line. The equation involves a nonlinear term with critical exponential growth in the Trudinger-Moser sense. We deal with periodic and asymptotically periodic potential which may change sign. We handle with the lack of compactness due to the unboundedness of the domain and the critical behavior of the nonlinearity. The main theorems are stated without the well known Ambrosetti-Rabinowitz condition at infinity.

1 Introduction

This work deals with the following class of fractional Kirchhoff equations

$$\left(a + b[u]_{1/2}^2\right)(-\Delta)^{1/2}u + V(x)u = f(x,u), \quad x \in \mathbb{R}, \tag{P}$$

where $a > 0$, $b \geq 0$, $(-\Delta)^{1/2}$ denotes the *square root of the Laplacian* and the term $[u]_{1/2}$ is the so-called *Gagliardo semi-norm* of the function u. We study the existence of bound and ground solutions for equation (P). It is worthwhile to mention that a solution u of finite energy for (P) is called a *bound state solution*. It is well known that $u \neq 0$ is called *ground state solution* if admits the smallest energy among the all nontrivial bound states of (P). This class of equations imposes several difficulties. The first one is the "lack of compactness" inherited by the nature of the problem by reason of equation (P) is defined in the whole space $\mathbb{R}$ and involves critical nonlinear terms. Furthermore, we have the presence of the term $b[u]_{1/2}^2$ that causes some mathematical difficulties and makes the study of such class of equations very interesting. Moreover, we deal with periodic functions and perturbations of periodic functions. We consider a class of bounded potentials which may change the sign. The nonlinear terms have critical exponential growth in the sense of Trudinger-Moser and are assumed without the well known Ambrosetti-Rabinowitz condition at infinity. In order to overcome these difficulties, we use a variational approach based on minimax theorems. To the best of our knowledge, there are few papers in the literature on fractional Kirchhoff equations involving critical exponential growth.

Remark 1.1. *If $a = 1$ and $b = 0$, then Problem (P) reduces to the fractional Schrödinger equation*

$$(-\Delta)^s u + V(x)u = f(x,u), \quad x \in \mathbb{R}^N,$$

which had been studied by many authors under many different assumptions on the potential $V(x)$ and nonlinearity $f(x,u)$. Here we improve the results obtained in [1], since we obtain ground state solutions and we are not assuming the Ambrosetti-Rabinowitz condition at infinity.

First, we are interested to study the following class of fractional Kirchhoff equations

$$\left(a + b[u]_{1/2}^2\right)(-\Delta)^{1/2}u + V_0(x)u = f_0(x,u), \quad x \in \mathbb{R}, \tag{P_0}$$

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where $V_0(x)$ and $f_0(x,u)$ are periodic functions on x and $V_0(x)$ satisfies the following assumptions:

(V_1) There exists a positive constant v_0 such that $V_0(x) \geq -v_0$ for all $x \in \mathbb{R}$;

(V_2) The infimum

$$\inf_{\substack{u\in E_0\\ \|u\|_2=1}} \left(\frac{a}{2\pi} \int_{\mathbb{R}^2} \frac{|u(x)-u(y)|^2}{|x-y|^2}\, \mathrm{d}x\mathrm{d}y + \int_{\mathbb{R}} V_0(x)u^2\, \mathrm{d}x \right)$$

is positive.

We are interested in the case that the nonlinear term has *critical exponential growth* and in addition, we suppose that $f_0(x,t)$ is a continuous 1-periodic function in x and satisfies the following hypotheses:

(f_1) $0 \leq \lim_{t\to 0} \dfrac{f_0(x,t)}{t} < \lambda_1$;

(f_2) $f_0 : \mathbb{R}\times\mathbb{R} \to \mathbb{R}$ is locally bounded in t, that is, for any bounded interval $J \subset \mathbb{R}$, there exists $C > 0$ such that $|f_0(x,t)| \leq C$, for all $(x,t) \in \mathbb{R} \times J$;

(f_3) $f_0(x,t)/|t|^3$ is strictly increasing for $t \in \mathbb{R}$, for all $x \in \mathbb{R}$;

(f_4) There exist constants $p > 4$ and $C_p > 0$ such that $f_0(x,t) \geq C_p t^{p-1}$, for all $(x,t) \in \mathbb{R}\times\mathbb{R}$.

2 Main Results

Theorem 2.1. *Assume that* (V_1), (V_2), (f_1)-(f_4) *hold and* f_0 *has critical exponential growth. Then problem* (P_0) *has a nontrivial bound state solution for some* $C_p > 0$ *large enough.*

We are also concerned with the existence of solutions to the class of fractional Kirchhoff equations given in (P). In this case in order to deal with the difficulties imposed by the lack of periodicity, we require assumptions which compare the periodic terms with the asymptotically periodic terms. On the potential $V(x)$ and on the nonliner term we assume that:

(V_3) $V_0 - V \in \mathcal{F}$ and $V_0(x) \geq V(x) \geq -v_0$, for all $x \in \mathbb{R}$;

(V_4) The infimum

$$\inf_{\substack{u\in E\\ \|u\|_2=1}} \left(\frac{a}{2\pi} \int_{\mathbb{R}^2} \frac{|u(x)-u(y)|^2}{|x-y|^2}\, \mathrm{d}x\mathrm{d}y + \int_{\mathbb{R}} V(x)u^2\, \mathrm{d}x \right)$$

is positive.

(f_5) For any $\varepsilon > 0$, there exists $\eta > 0$ such that for $t \in \mathbb{R}$ and $|x| \geq \eta$ we have

$$|f(x,t) - f_0(x,t)| \leq \varepsilon \mathrm{e}^{\alpha t^2}.$$

Moreover, $f_0(x,t) \leq f(x,t)$ for all $(x,t) \in \mathbb{R}\times\mathbb{R}$.

Theorem 2.2. *Assume that the potential* $V(x)$ *satisfies* (V_1)-(V_4). *If the nonlinearity* f *satisfies* (f_1)-(f_5) *and has critical exponential growth then Problem* (P) *has a ground state solution for some* $C_p > 0$ *large enough.*

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