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Regularity up to the Boundary for solutions of Fully Nonlinear Ellipt Equations

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Abstract

We provide regularity results at the boundary for viscosity solutions to second order fully nonlinear uniformly elliptic equations in the form $F(D^2u(x), Du(x), x) = f(x)$ in Ω even in the case when the source function lies in the borderline cases of the theory.

1 Introduction

We know that it is possible to obtain regularity up to the boundary, putting toguether interior regularaity and boundary regularity if the last ones are available. So, thanks to the recent developments obtained by Teixeira [1] for fully nonlinear second-order uniformly elliptic equations of the form

$$F(D^2u(x), x) = f(x) \quad \text{em} \quad \Omega, \tag{RI}$$

where it was obtained interior regularity for viscosity solutions, the appropriate weak notion of solutions for (RI) and boundary regularity estimates available in the literature, see for example [2], [3], [4] and [5], just to cite a few, we can obtain the following glogal regularity results. summarized in the following table

2 Main Results

We have to extend the interior estimates obtained by Teixeira for solutions to gradient dependent equations of the form

$$F(D^2u(x), Du(x), x) = f(x) \quad \text{em} \quad \Omega. \tag{RI}$$

So, this is our fist difficulty. In fact, we are going to flatten the domain Ω by a change of variables to $B_1^+ := \{x \in B_1 : x_n > 0\}$ and gradient terms are unavoidable for the change of variables. So, we have to suppose Ω a smooth domain (C^2 is sufficient).

We are going to present the following regularity estimates for solutions of

$$\left\{ \begin{array}{rcll} F(D^2u(x), Du(x), x) & = & f(x) & \text{em} \quad \Omega \\ u & = & \varphi & \text{sobre} \quad \partial\Omega \end{array} \right. \tag{GR}$$

summarized in the following table: where $alpha_0$ is the universal optimal exponent for solutions of the respective

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homogeneous, constant coefficient case (the *tangential problem*) and ε is the escauriaza constant (see [Es93]). It is worth nothing that in the case $f \in BMO$ we need $C^{2,\varepsilon}$ a priori estimates for the Tangential Problem (the respective, homogeneous contant coefficient case), since $C^{1,Log-Lip}$ is more regular than the Tangential Problem that is merely $C^{1,\alpha}$.

References

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