Analysis of PDEs: Theory and applications

Summer School 2024 - PPGMAT UFPB

January 8th to March 19th

Title and abstracts

Nehari method on cones

Denilson da Silva Pereira
Federal University of Campina Grande (Brazil)

In this talk we develop an abstract theory of the Nehari method on cones. We apply our method to several elliptic problems involving p-laplacian and Kirchhoff operator as well as in some classes of problems arising in population dynamics.

This is a joint work with João Rodrigues (UFPA) and Felipe Silva (UFPA).

Existence of bound states for quasilinear elliptic problems involving critical growth and frequency

Diego Ferraz
Federal University of Rio Grande do Norte (Brazil)

In this paper we study the existence of bound states for the following class of quasilinear problems $-\varepsilon^p \Delta_p u + V(x)u^{p-1} = f(u) + u^{p^*-1}$,$u > 0$ in $\mathbb{R}^N$, $\lim_{|x|\to\infty} u(x) = 0$ where $\varepsilon > 0$ is small, $1 < p < N$, f is a nonlinearity with general subcritical growth in the Sobolev sense, $p^* = pN/(N-p)$ and V is a continuous nonnegative potential. By introducing a new set of hypotheses, our analysis includes the critical frequency case which allows the potential V to not be necessarily bounded below away from zero. We also study the regularity and behavior of positive solutions as $|x| \to \infty$ or $\varepsilon \to 0$, proving that they are uniformly bounded and concentrate around suitable points of $\mathbb{R}^N$, that may include local minima of V.

Singular solutions to k-Hessian equations with fast-growing nonlinearities

Esteban da Silva
Federal University of Rio Grande do Norte (Brazil)

In this talk we present recent advances on the study of a class of elliptic problems, involving a k-Hessian and a very fast-growing nonlinearity, on a unit ball. We prove the existence of a radial singular solution and obtain its exact asymptotic behavior in a neighborhood of the origin. Furthermore, we study the multiplicity of regular solutions and bifurcation diagrams. An important ingredient of this study is analyzing the number of intersection points between the singular and regular solutions for rescaled problems. In the particular case

of the exponential nonlinearity, we obtain the convergence of regular solutions to the singular.

Joint work with: J. M. do Ó and E. Shamarova.

On nonlinear perturbations of a periodic integrodifferential Kirchhoff Equation with critical exponential growth

Eudes Mendes Barbosa
Federal Rural University of Pernambuco (Brazil)

In this talk, we investigate the existence of solutions for a class of integrodifferential Kirchhoff equations. These equations involve a nonlocal operator with a measurable kernel that satisfies "structural properties" that are more general than the standard kernel of the fractional Laplacian operator. Additionally, the potential can be periodic or asymptotically periodic, and the nonlinear term exhibits critical exponential growth in the sense of Trudinger–Moser inequality. To guarantee the existence of solutions, we employ variational methods, specifically the mountain-pass theorem. In this context, it is important to emphasize that we have additional difficulties due to the lack of compactness in our problem, because we deal with critical growth nonlinearities in unbounded domains. Moreover, the Kirchhoff term adds complexity to the problem, as it requires suitable calculations for control the estimate the minimax level, representing the main challenge in this work. Finally, we consider two different approaches to estimate the minimax level. The first approach is based on a hypothesis proposed by D. M. Cao, while the second one involves a slightly weaker assumption addressed by Adimurthi and Miyagaki.

Rupture solutions in MEMS problems

Evelina Shamarova
Federal University of Paraíba (Brazil)

In this talk, we discuss general equations modeling electrostatic MEMS devices

$$\begin{cases} \varphi\big(r, -u'(r)\big) = \lambda \int_0^r \frac{f(s)}{g(u(s))}\, ds, & r \in (0,1), \\ 0 \le u(r) \le 1, & r \in (0,1), \\ u(1) = 0, \end{cases} \qquad (P_\lambda)$$

where φ, g, f are continuous functions on $(0,1)$ and $\lambda > 0$ is a parameter. We obtain results on the existence and regularity of a touchdown solution to (P_λ) and find upper and lower bounds for the respective pull-in voltage. In the particular case, when $\varphi(r,v) = r^\alpha |v|^\beta v$, i.e., when the associated differential equation involves the operator $-r^{-\gamma}(r^{\alpha}|u'|^{\beta}u')'$, we obtain the exact asymtotic behavior of the touchdown solution in a neighborhood of the origin.

Joint work with: J. M. do Ó, R. Clemente and E. da Silva.

Some weighted Sobolev space and applications

Everaldo Souto de Medeiros
Federal University of Paraíba (Brazil)

In this presentation, we introduce a new Hardy-Sobolev inequality in the upper-half space. As a consequence, we derive some new weighted Sobolev spaces. These embedding results are then applied to address certain elliptic equations in the upper-half space.

Schrödinger-Poisson system with zero mass in $\mathbb{R}^2$ involving $(2, q)$-Laplacian

José Carlos de Albuquerque
Federal University of Pernambuco (Brazil)

In this talk we discuss the existence of positive least energy solution for a class of planar elliptic systems in the zero mass case. Due to the nature of the problem, we deal with the logarithmic integral kernel. Furthermore, we discuss the asymptotic behavior and regularity of the solutions.

Joint work with: J. Carvalho and E. Silva.

Existence and Multiplicity of Positive Solutions for a Quasilinear Problem with nonhomogeneous Dirichlet condition

Leonelo Iturriaga
Federico Santa María Technical University (Chile)

In this talk, we study the existence and multiplicity of solutions for the following quasilinear elliptic problem:

$$\begin{cases} -\Delta_p u = f(x,u) & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = \varphi & \text{on } \partial\Omega. \end{cases} \tag{1}$$

Here Ω is a bounded and smooth domain in $\mathbb{R}^N$, $p \in (1, N)$ and $0 \leq \varphi \in C^{1,\alpha}(\partial\Omega)$. f is locally "superlinear" at infinity, it may change sign, and it may not satisfy the (AR)-type condition. To address this problem, we mainly use variational approach together with the upper-lower solutions method and a variant of the Mountain Pass theorem due to Schechter.

Joint work with: F. Perez

Trudinger–Moser type inequalities with a symmetric conical metric and a symmetric potential

Manassés Xavier de Souza

Federal University of Paraíba (Brazil)

In this talk, we will study some improvements related to the Trudinger-Moser inequality involving a symmetric conic metric on a closed Riemann surface. Additionally, we employ the method of blow-up analysis to derive the corresponding extremals.

Schrödinger equations involving weights and potentials with a type of pseudo-coercivity

Pedro Ubilla

University of Santiago, Chile

We establish the existence of weak solutions to a nonlinear Schrödinger equation given by $-\mathrm{div}(a(x)\nabla u)+V(x)u = \lambda f(x,u)$ in $\mathbb{R}^N$, where λ is a positive parameter and f, a, and V are continuous functions. Our approach allow us to consider an innovation because we may assume that $x \mapsto a(x)+V(x)$ is coercive, without necessarily $a(x)$ and $V(x)$ being. In addition, we do not impose any assumptions on the nonlinearity f at ∞. We establish our results through a combination of truncation techniques, a priori estimates, and variational methods.

Existence of solutions for a fractional Choquard-type equation in $\mathbb{R}$ with critical exponential growth

Rodrigo Clemente

Federal Rural University of Pernambuco (Brazil)

In this paper we study the following class of fractional Choquard–type equations

$$(-\Delta)^{1/2}u + u = (I_\mu * F(u))\, f(u), \quad x \in \mathbb{R},$$

where $(-\Delta)^{1/2}$ denotes the 1/2–Laplacian operator, I_μ is the Riesz potential with $0 < \mu < 1$ and F is the primitive function of f. We use Variational Methods and minimax estimates to study the existence of solutions when f has critical exponential growth in the sense of Trudinger–Moser inequality.

Compactness of singular solutions to higher order GJMS equations

João Henrique Andrade

Federal University of Paraíba (Brazil)

We study some compactness properties of the set of conformally flat singular metrics with constant, positive higher order Q-curvature on a finitely punctured sphere. Building on the classification of the local asymptotic behavior near isolated singularities, we introduce a notion of necksize for these metrics in our moduli space, which we use to characterize compactness. More precisely, we prove that if the punctures remain separated and the necksize at each puncture is bounded away from zero along a sequence of metrics, then a subsequence converges with respect to the Gromov–Hausdorff metric. Our proof relies on an upper bound estimates proved using moving planes and a blow-up argument. This is combined with a lower bound estimate which is a consequence of a removable singularity theorem. We also introduce a higher order homological invariant which may be of independent interest for upcoming research.

Trudinger-Moser and Adams type inequalities on weighted Sobolev spaces and applications

Raoní Ponciano

Federal University of Paraíba (Brazil)

In this talk, we will explore sharp Sobolev embeddings and Adams-type inequalities on a class of Sobolev spaces with potential weights, without assuming boundary conditions. We will focus on the borderline Sobolev embedding into the exponential class, highlighting its sharp constant. By applying these concepts, we will demonstrate the existence of nontrivial solutions for elliptic equations with nonlinearities including both polynomial and exponential growths. Additionally, we will investigate embeddings on a class of Sobolev spaces with potential weights on unbounded domains. Our findings will reveal embeddings into weighted Lebesgue spaces, specifically L^q_θ, using radial power weights. Based on these embeddings, we will investigate

the existence and non-existence of maximizers for Trudinger-Moser type inequalities. Furthermore, we will present a technique to enhance the maximal integrability by selectively removing necessary terms from the exponential series while maintaining the continuity of the embedding.

Coupled Elliptic systems with sublinear growth

Juan Luis Arratia
University of Santiago, Chile

Consider the coupled elliptic system

$$\begin{cases} -\Delta u + u = \rho_1(x)u^{p_1} + \lambda v & \text{in } \mathbb{R}^N \\ -\Delta v + v = \rho_2(x)v^{p_2} + \lambda u & \text{in } \mathbb{R}^N, \\ u(x), v(x) \to 0 & \text{as } |x| \to \infty. \end{cases}$$

We observe that in [Ambrosetti A, Cerami G. and Ruiz D., 2008] it was proved the existence of positive bound and ground states in the case $\lambda \in (0,1)$, $p_1 = p = p_2$, $1 < p < 2^* - 1$, $\rho_1(x)$ and $\rho_2(x)$ tends to one at infinity. In this work we complement their result, because we show that the previous system has no solutions when $0 < p_1, p_2 < 1$, as well as we establish sharp hypotheses on the powers $0 < p_1, p_2$ the parameter λ and the weights $\rho_1(x)$, $\rho_2(x)$ that will allow us to obtain the existence and uniqueness of a positive bounded solution. This is a joint work with P. Ubilla (USACH).

Período de 18 e 19 de março

Uniform Lipschitz estimates on the interface for solutions to two-phase free boundary problems governed by non-uniformly elliptic operator

Jefferson Abrantes

Federal University of Campina Grande (Brazil)

We will deal with a two-phase free boundary problem involving a degenerate non-uniformly elliptic operator with Phi-Laplacian type growth. We prove Lipschitz regularity for minimizers by controlling the negative phase density along the free boundary. It is also shown that the region where the local Lipschitz regularity fails is contained in the contact set between the positive and negative free boundaries and there the negative phase is cusp free.

This a joint work with Sergio Henrique Monari (ICMC-USP).
