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QUASILINEAR SCHRÖDINGER EQUATIONS INVOLVING CONCAVE AND CONVEX NONLINEARITIES

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Abstract. In this paper the mountain–pass theorem and the Ekeland variational principle in a suitable Orlicz space are employed to establish the existence of positive standing wave solutions for a quasilinear Schrödinger equation involving a combination of concave and convex terms. The second order nonlinearity considered in this paper corresponds to the superfluid equation in plasma physics.

1. **Introduction.** In this paper we are concerned with *quasilinear Schrödinger equations* of the form

$$i\frac{\partial\psi}{\partial t} = -\Delta\psi + W(x)\psi - \eta(|\psi|^2)\psi - \kappa\left[\Delta\rho(|\psi|^2)\right]\rho'(|\psi|^2)\psi, \tag{1}$$

where $\psi : \mathbb{R}\times\mathbb{R}^N \to \mathbb{C}$, κ is a positive constant, $W : \mathbb{R}^N \to \mathbb{R}$ is a given potential and $\rho,\ \eta : \mathbb{R}^+ \to \mathbb{R}$ are suitable functions. Quasilinear equations of the form (1) appear naturally in mathematical physics and have been derived as models of several physical phenomena corresponding to various types of nonlinear terms ρ. For more details see [15], [20] and references therein.

Here we consider the case where $\rho(s) = s$, $\kappa > 0$ and our special interest is in the existence of *standing wave solutions*, that is, solutions of type

$$\psi(t, x) = \exp(-iEt)u(x),$$

where $E \in \mathbb{R}$ and $u > 0$ is a real function. It is well known that ψ satisfies (1) if and only if the function $u(x)$ solves the following equation of elliptic type with the formal variational structure

$$-\Delta u + V(x)u - \kappa\left[\Delta\left(u^2\right)\right]u = \eta(u), \quad u > 0, \quad x \in \mathbb{R}^N, \tag{2}$$

where $V(x) := W(x) - E$ is the new potential, η is the new nonlinearity and without loss of generality we assume $\kappa = 1$.

We were motivated by several recent mathematical studies on the existence of solutions for (2). Among others we refer to [2], [9], [10], [12], [15], [16], [17], [19]

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