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# $2^a$ Prova: Cálculo Vetorial e Geometria Analítica

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**1 (4,0 pts.)** *dadas as retas* $r_1 : \begin{cases} x &= 3+s \\ y &= 1+s \\ z &= 4+2s \end{cases}$ *e* $r_2 : \begin{cases} x &= -1+3t \\ y &= 1-t \\ z &= -1+2t \end{cases}$ $s,\, t \in \mathbb{R}$.

(a) *Verifique que* $r_1$ *e* $r_2$ *são retas reversas;*

(b) *calcule* $d(r_1,\, r_2)$, *a distância entre* $r_1$ *e* $r_2$;

(c) *Determine as equações paramétricas da reta* $r$ *que interseta perpendicularmente as retas* $r_1$ *e* $r_2$.

**2 (3,0 pts.)** *Determine a equação cartesiana do plano* $\pi$ *descrito nos casos abaixo.*

(a) $\pi$ : *perpendicular ao vetor* $\vec{v} = [2, -2, 1]$ *cuja distância ao ponto* $P = (3, 2, -1)$ *é 2 unidades;*

(b) $\pi$ : *contém os pontos* $A = (1, 1, 0)$, $B = (0, 3, 0)$ *e* $C = (2, -1, 2)$;

(c) $\pi$ : *contem a reta* $r$ *de equação* $r : \begin{cases} x &= -1+2t \\ y &= 2+t \\ z &= 3-t \end{cases}$ $t \in \mathbb{R}$

*e é paralelo ao vetor* $[-2, 1, 3]$

**3 (2,0 pts.)** *Determine as equações paramétricas das seguintes retas:*

(a) *que passa pelo ponto* $A = (1, -2, -1)$ *e é perpendicular ao plano* $x - 2y + 3z = 5$;

(b) *que passa pelos pontos* $A = (1, 0, 2)$ *e* $B = (0, 2, -1)$.

**4 (2,0 pts.)** *Determine a distância do ponto* $P$ *à reta* $r$, *onde:*

$$P = (2, -1, 1) \qquad r : \frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-4}{3}.$$

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(1)

$$r_1 : \begin{cases} x = 3 + s \\ y = 1 + s \\ z = 4 + 2s \end{cases} \qquad r_2 : \begin{cases} x = -1 + 3t \\ y = 1 - t \\ z = -1 + 2t \end{cases}, \quad s, t \in \mathbb{R}$$

(a) $r_1$ e $r_2$ são retas reversas.

$\vec{v_1} = [1, 1, 2] \parallel r_1$, $\vec{v_2} = [3, -1, 2] \parallel r_2$

$\vec{v_1} \nparallel \vec{v_2}$ logo $r_1 \nparallel r_2$

$P_1 = (3, 1, 4) \in r_1$, $P_2 = (-1, 1, -1) \in r_2$

$\overrightarrow{P_1P_2} = [-4, 0, -5]$

Os vetores $\vec{v_1}$, $\vec{v_2}$ e $\overrightarrow{P_1P_2}$ são coplanares?

$$\det \begin{pmatrix} 1 & 1 & 2 \\ 3 & -1 & 2 \\ -4 & 0 & -5 \end{pmatrix} = 5 + 0 - 8 - 8 - 0 + 15 = 4 \neq 0 \ldots (*)$$

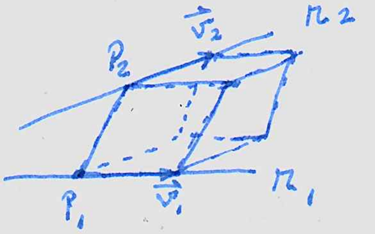
$\therefore$ $\vec{v_1}$, $\vec{v_2}$ e $\overrightarrow{P_1P_2}$ não são coplanares e logo as retas $r_1$ e $r_2$ não se intersetam

Assim, $r_1 \nparallel r_2$ e $r_1 \cap r_2 = \emptyset$

$\therefore$ $r_1$ e $r_2$ são retas reversas

(b) $d(r_1, r_2) = ?$

$$d(r_1, r_2) = \frac{|\vec{v_1} \times \vec{v_2} \cdot \overrightarrow{P_1P_2}|}{\|\vec{v_1} \times \vec{v_2}\|}$$

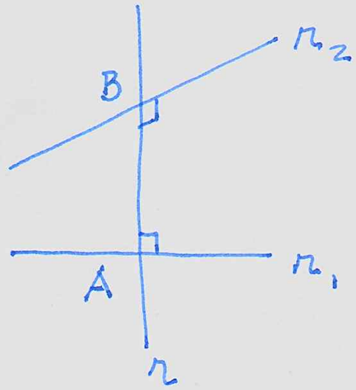



$$|\vec{v_1} \times \vec{v_2} \cdot \overrightarrow{P_1P_2}| = |4| = 4 \quad \left(\begin{array}{c}\text{veja } * \\ \text{acima}\end{array}\right)$$

$$\vec{v_1} \times \vec{v_2} = [4, 4, -4], \quad \|\vec{v_1} \times \vec{v_2}\| = \sqrt{16 + 16 + 16} = 4\sqrt{3}$$

$$d(r_1, r_2) = \frac{4}{4\sqrt{3}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

(c)




$\{A\} = r \cap r$

$\{B\} = r \cap r$

$$A = (3+s, 1+s, 4+2s), \quad B = (-1+3t, 1-t, -1+2t)$$

$$\overrightarrow{AB} = [-1+3t-3-s, \cancel{1}-t-\cancel{1}-s, -1+2t-4-2s]$$

$$\overrightarrow{AB} = [3t-s-4, -t-s, 2t-2s-5]$$

$$\overrightarrow{AB} \perp \vec{v}_1 \Rightarrow (3t-s-4)+(-t-s)+2(2t-2s-5) = 0$$

$$\Rightarrow 6t-6s = 14 \qquad \boxed{3t-3s=7}$$

$$\overrightarrow{AB} \perp \vec{v}_2 \Rightarrow 3(3t-s-4)-(-t-s)+2(2t-2s-5) = 0$$

$$\Rightarrow 14t-6s = 22 \qquad \boxed{7t-3s=11}$$

$$\begin{array}{l} 3t-3s=7 \\ 7t-3s=11 \end{array} \qquad 4t = 4 \Rightarrow \boxed{t=1}$$

$$t=1 \Rightarrow \boxed{s = -4/3}$$

$$t=1 \Rightarrow B = (2, 0, 1)$$

$$s = -4/3 \Rightarrow A = (3-4/3, 1-4/3, 4-8/3)$$

$$\Rightarrow A = (5/3, -1/3, 4/3)$$

$$r: \begin{cases} x = 2 + \alpha \\ y = 0 + \alpha \\ z = 1 - \alpha \end{cases}, \quad \alpha \in \mathbb{R}$$

$$\left( \vec{AB} = [1/3, 1/3, -1/3] \parallel [1,1,-1] \right)$$

(2) Eq. cartesiana do plano $\pi$

(a) $\pi \perp \vec{v}$, $\vec{v} = [2,-2,1]$

$d(P,\pi) = 2$, $P = (3,2,-1)$

$$\vec{v} \perp \pi \Rightarrow \pi : 2x - 2y + z = d$$

$$d(P,\pi) = 2 \Rightarrow \frac{|2\cdot 3 - 2\cdot 2 - 1 - d|}{\sqrt{4+4+1}} = 2$$

$$\Rightarrow \frac{|1-d|}{3} = 2$$

$$\Rightarrow |1-d| = 6$$

$1 - d = 6 \Rightarrow \underline{d = -5}$

$1 - d = -6 \Rightarrow \underline{d = 7}$

~~$\pi: 2x-2y$~~ $\pi: 2x - 2y + z = -5$

ou $\pi: 2x - 2y + z = 7$

(b) $A, B, C \in \pi$ , $A = (1,1,0)$ , $B = (0,3,0)$ e
$C = (2,-1,2)$

$$\overrightarrow{AB} = [-1,2,0]$$
$$\overrightarrow{AC} = [1,-2,2]$$

( $\overrightarrow{AB} \nparallel \overrightarrow{AC}$ , portanto os ptos A, B e C não são colineares )

$$\overrightarrow{AB} \times \overrightarrow{AC} = [4,2,0]$$

$\pi: 4x + 2y = 6$ $\qquad \boxed{2x + y = 3}$

(c) $r \subseteq \pi$

$\vec{v} = [-2,1,3] \parallel \pi$

$$r: \begin{cases} x = -1 + 2t \\ y = 2 + t \\ z = 3 - t \end{cases}, \quad t \in \mathbb{R}$$

$\vec{u} = [2,1,-1] \parallel r$ $\qquad P = (-1,2,3) \in r$

$$r \subseteq \pi \implies \vec{u} \parallel \pi$$

$$\vec{u} \parallel \pi, \ \vec{v} \parallel \pi \implies \vec{u} \times \vec{v} \perp \pi.$$

$$\vec{u} \times \vec{v} = [-4,4,-4] \perp \pi$$

$$\pi: \quad -4x + 4y - 4z = -4(-1) + 4 \cdot 2 - 4 \cdot 3 = 0$$

$$\pi: -4x + 4y - 4z = 0$$

ou

$$\boxed{\pi: x - y + z = 0}$$

③ Eq. paramétrica da reta $r$

(a) $A \in r$ $\quad A = (1,-2,-1)$

$r \perp \pi$ $\quad \pi: x - 2y + 3z = 5$

$\vec{n} = [1,-2,3] \perp \overline{\overline{\pi}} \quad \therefore \quad \vec{n} \parallel r \quad (r \perp \pi)$

$$r: \begin{cases} x = 1 + t \\ y = -2 - 2t \\ z = -1 + 3t \end{cases}, \ t \in \mathbb{R}$$

(b) $A, B \in r$ $\quad A = (1,0,2)$, $B = (0,2,-1)$

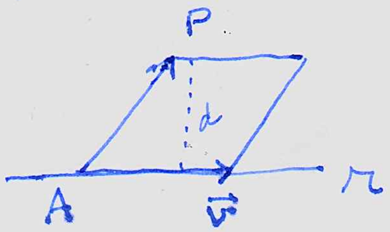
$\vec{AB} \parallel r$, $\quad \vec{AB} = [-1,2,-3]$

$$r: \begin{cases} x = 1 - t \\ y = 0 + 2t \\ z = 2 - 3t \end{cases}, \ t \in \mathbb{R}$$

④ $d(P, r) = ?$

$P = (2,-1,1)$, $\quad r: \dfrac{x-1}{2} = \dfrac{y+3}{-1} = \dfrac{z-4}{3}$

$\left( \dfrac{2-1}{2} \neq \dfrac{-1+3}{-1} \quad \therefore \quad P \notin r \right)$




$\vec{v} = [2,-1,3] \parallel r$

$A = (1,-3,4) \in r$

$$d(P, r) = \frac{\| \vec{v} \times \vec{AP} \|}{\| \vec{v} \|}$$

$\vec{v} = [2, -1, 3] \quad \Rightarrow \quad \| \vec{v} \| = \sqrt{4 + 1 + 9} = \sqrt{14}$

$\vec{AP} = [+1, 2, -3]$

$\vec{v} \times \vec{AP} = [-3, 9, 5] \qquad \| \vec{v} \times \vec{AP} \| = \sqrt{9 + 81 + 25}$

$= \sqrt{115}$

$$\therefore \quad d(P, r) = \frac{\sqrt{115}}{\sqrt{14}} = \sqrt{\frac{115}{14}}$$