C3 **LISTA 4**

① Calcular $\int_C (xy + y + z)\, ds$

$C: \quad \alpha(t) = 2t\vec{i} + t\vec{j} + (2-2t)\vec{k}, \quad 0 \leq t \leq 1$

Solução

$$\int_C f\, ds = \int_a^b f(\alpha(t))\, \|\alpha'(t)\|\, dt.$$

$$f(\alpha(t)) = 2t \cdot t + t + 2 - 2t = 2t^2 - t + 2$$

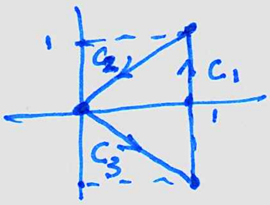
$$\alpha'(t) = 2\vec{i} + \vec{j} - 2\vec{k}, \quad \|\alpha'(t)\| = \sqrt{4+1+4} = \sqrt{9} = 3$$

$$\int_C f\, ds = \int_0^1 (2t^2 - t + 2) \cdot 3\, dt = \int_0^1 (6t^2 - 3t + 6)\, dt$$

$$= \left(2t^3 - \frac{3t^2}{2} + 6t\right)\Big|_0^1 = 2 - \frac{3}{2} + 6 = 13/2$$

② $\int_C (x+y)\, ds$, C é o triângulo de vértices $(0,0)$, $(1,1)$ e $(-1,1)$

Solução



$C = C_1 \cup C_2 \cup C_3$

$C_1: \begin{cases} x = 1 \\ -1 \leq y \leq 1 \end{cases}$

$\alpha_1(t) = (1-t)(1,-1) + t(1,1)$

$\begin{cases} \alpha_1(t) = (1, -1+2t) \\ 0 \leq t \leq 1 \end{cases}$

$C_2: \ \alpha_2(t) = (1-t)(1,1) \quad t \in [0,1]$

$\alpha_2(t) = (1-t, 1-t), \quad t \in [0,1]$

$C_3: \ \alpha_3(t) = (t, -t), \quad t \in [0,1]$

$$\int_C f\, ds = \sum_{j=1}^{3} \int_{C_j} f\, ds.$$

$$\int_{C_1}(x+y)\,ds = \int_0^1 (1+(-1)+2t)\cdot 2\,dt = \int_0^1 4t\,dt = 2t^2\Big|_0^1 = 2$$

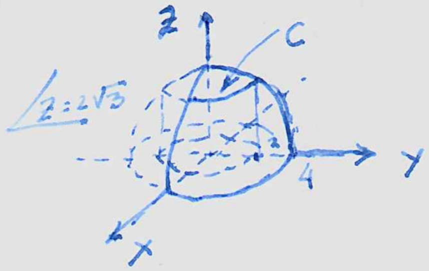
$$\int_{C_2}(x+y)\,ds = \int_0^1 2(1-t)\sqrt{2}\,dt = (2\sqrt{2}\,t - \sqrt{2}\,t^2)\Big|_0^1 = 2\sqrt{2}-\sqrt{2} = \sqrt{2}$$

$$\int_{C_3}(x+y)\,ds = \int_0^1 (t-t)\sqrt{2}\,dt = 0$$

$$\therefore \quad \int_C (x+y)\,ds = 2+\sqrt{2}$$

③ Calcular $\int_C \sqrt{3}\,xyz\,ds$. $\quad C: \begin{cases} x^2+y^2+z^2=16 \\ x^2+y^2=4 \\ x\geq 0,\ y\geq 0,\ z\geq 0 \end{cases}$

Solução

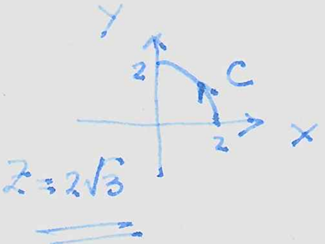


$$\left.\begin{array}{l} x^2+y^2=4 \\ 4+z^2=16 \\ z^2=12 \end{array}\right\} \quad z = \pm 2\sqrt{3}$$

$$C: \begin{cases} z = 2\sqrt{3} \\ x^2+y^2=4 \\ x\geq 0,\ y\geq 0 \end{cases}$$

Uma parametrização para C:

$$\gamma(t) = (2\cos t, 2\,\mathrm{sen}\,t, 2\sqrt{3}), \quad 0 \leq t \leq \pi/2$$



$$\begin{cases} xyz = 8\sqrt{3}\,\mathrm{sen}\,t\cos t \\ \gamma'(t) = (-2\,\mathrm{sen}\,t, 2\cos t, 0) \\ \|\gamma'(t)\| = 2 \end{cases}$$

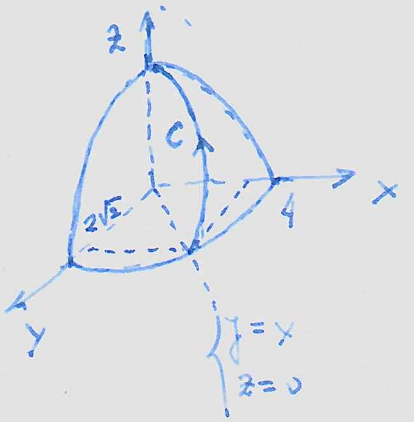
$$\int_C \sqrt{3}\,xyz\,ds = \int_0^{\pi/2} 8\sqrt{3}\cdot\sqrt{3}\,\mathrm{sen}\,t\cos t\cdot 2\,dt$$

$$= 48\int_0^{\pi/2} \mathrm{sen}\,t\cos t\,dt = 48\cdot\frac{1}{2}\,\mathrm{sen}^2 t\Big|_0^{\pi/2}$$

$$\int_C \sqrt{3}\, xyz \, ds = 24 \,\mathrm{sen}^2 t \Big|_0^{\pi/2} = 24(1-0) = 24$$

④ Calcular $\displaystyle\int_C (x^2+y^2)\,ds$ $\quad C: \begin{cases} x^2+y^2+z^2 = 16 \\ y = x \\ x \geq 0,\ y \geq 0,\ z \geq 0 \end{cases}$

Solução



$$x^2 + x^2 + z^2 = 16$$
$$2x^2 + z^2 = 16$$
$$\frac{x^2}{8} + \frac{z^2}{16} = 1$$

$$\begin{cases} \left(\dfrac{x}{2\sqrt{2}}\right)^2 + \left(\dfrac{z}{4}\right)^2 = 1 \\ y = x \end{cases}$$

Uma parametrização para a curva C:

$$\gamma(t) = \left(2\sqrt{2}\cos t,\ 2\sqrt{2}\cos t,\ 4\,\mathrm{sen}\, t\right)$$
$$0 \leq t \leq \pi/2$$

$$\int_C (x^2+y^2)\,ds = \int_0^{\pi/2} 16\cos^2 t \cdot 4\,dt$$

$$\begin{cases} \gamma' = \left(-2\sqrt{2}\,\mathrm{sen}\, t,\ -2\sqrt{2}\,\mathrm{sen}\, t,\ 4\cos t\right) \\ \|\gamma'\| = 4 \end{cases}$$

$$= 16 \int_0^{\pi/2} \frac{1+\cos 2t}{2}\,dt$$

$$= 8 \cdot \pi/2 + 8 \cdot \frac{1}{2}\,\mathrm{sen}\, 2t \Big|_0^{\pi/2} = 4\pi + 4(0-0)$$

$$= 4\pi$$

5

$$C: \begin{cases} x^2+y^2+z^2=5 \\ z \geq 0 \\ x+y=1 \end{cases} \qquad I_z = ?$$

densidade : proporcional à distância ao plano XY

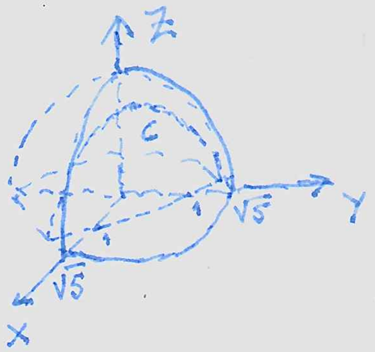
$(\ f(x,y,z) = kz\ )$

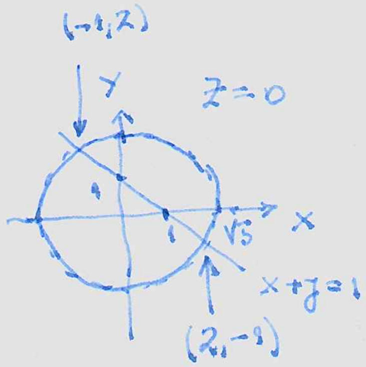
Solução:

$$I_z = k\int_C d^2 \cdot f \, ds$$

d = distância ao eixo Z
$= \sqrt{x^2+y^2}$
$\therefore \ d^2 = x^2+y^2$

$$\boxed{I_z = k\int_C (x^2+y^2)\, z \, ds}$$





$y = 1-x$

$x^2+(1-x)^2+z^2=5$

$$\left\{ \begin{array}{l} 2x^2-2x+z^2=4 \\ 2(x^2-x)+z^2=4 \\ 2\left(x-\frac{1}{2}\right)^2+z^2=4+\frac{1}{2} \\ 2\left(x-\frac{1}{2}\right)^2+z^2=\frac{9}{2} \\ \left(\dfrac{x-1/2}{3/2}\right)^2+\left(\dfrac{z}{3/\sqrt{2}}\right)^2=1 \end{array} \right.$$

$$\frac{x-1/2}{3/2} = \cos t$$

$$\frac{z}{3/\sqrt{2}} = \operatorname{sen} t$$

$(y = 1-x)$

$$\gamma: \begin{cases} x = \frac{1}{2}+\frac{3}{2}\cos t \\ y = \frac{1}{2}-\frac{3}{2}\cos t \qquad 0 \leq t \leq \pi \\ z = \frac{3}{\sqrt{2}}\operatorname{sen} t \end{cases}$$

$$\left.\begin{array}{l} x=2 \\ y=-1 \\ z=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \cos t = 1 \\ \operatorname{sen} t = 0 \end{array}\right\} \Rightarrow t = 0$$

$$\left.\begin{array}{l} x=-1 \\ y=2 \\ z=0 \end{array}\right\} \Rightarrow \left.\begin{array}{l} \cos t = -1 \\ \operatorname{sen} t = 0 \end{array}\right\} \Rightarrow t = \pi$$

$$x^2+y^2 = x^2+(1-x)^2 = 2x^2-2x+1 = 2(x-1/2)^2+1/2$$

$$x^2+y^2 = 2\cdot\frac{9}{4}\cos^2 t + 1/2$$

$$(x^2+y^2)z = \left(\frac{9}{2}\cos^2 t + 1/2\right)\frac{3}{\sqrt{2}}\,\text{sen}\,t$$

$$(x^2+y^2)z = \frac{27}{2\sqrt{2}}\cos^2 t\,\text{sen}\,t + \frac{3}{2\sqrt{2}}\,\text{sen}\,t$$

$$\gamma' : \begin{cases} x' = -\frac{3}{2}\,\text{sen}\,t \\ y' = 3/2\,\text{sen}\,t \\ z' = \frac{3}{\sqrt{2}}\cos t \end{cases} \qquad \|\gamma'\| = \sqrt{9/2} = \frac{3}{\sqrt{2}}$$

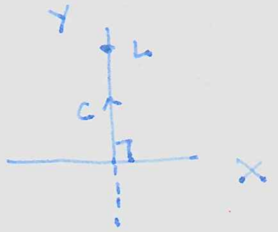
$$I_z = k\int_C (x^2+y^2)z\,ds = k\int_0^{\pi}\left(\frac{27}{2\sqrt{2}}\cos^2 t\,\text{sen}\,t + \frac{3}{2\sqrt{2}}\,\text{sen}\,t\right)\cdot\frac{3}{\sqrt{2}}\,dt$$

$$= k\,\frac{81}{4}\left(-\frac{1}{3}\cos^3 t\right)\Big|_0^{\pi} + k\,\frac{9}{4}(-\cos t)\Big|_0^{\pi}$$

$$= \frac{27}{2}k + \frac{9}{2}k = \frac{36}{2}k$$

$$= 18k$$

6



$I_x = ?$ $\quad C: \gamma(t) = (0, t)$, $0 \le t \le L$

$\gamma' = (0,1)$, $\|\gamma'\| = 1$

$$I_x = \int_0^L y^2\cdot k\,ds = \int_0^L k\,t^2\,dt = \frac{k}{3}t^3\Big|_0^L$$

$$= \frac{k}{3}L^3$$

7

$$C: \begin{cases} x^2+y^2+z^2=16, & x\geq 0 \\ y+z=4 \end{cases}$$

densidade: $f(x,y,z)=x$

$I_x = \frac{32}{3} M$?

M = massa do arame (c)

Solução

$$I_x = \int_C (y^2+z^2)\cdot x \, ds$$

$$\boxed{y = 4-z}$$

$$x^2+(4-z)^2+z^2=16$$

$$x^2-8z+2z^2=0$$

$$x^2+2(z^2-4z)=0$$

$$x^2+2(z-2)^2=8$$

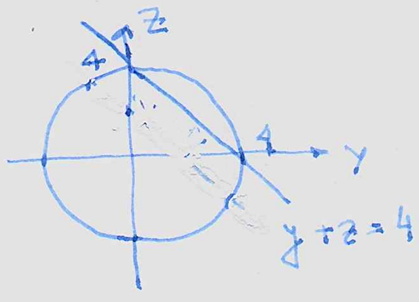
$$\frac{x^2}{8}+\frac{(z-2)^2}{4}=1$$

$$\left(\frac{x}{2\sqrt{2}}\right)^2+\left(\frac{z-2}{2}\right)^2=1$$

$$\gamma: \begin{cases} x = 2\sqrt{2}\cos t \\ y = 4-z = 2-2\,\text{sen}\, t \\ z = 2+2\,\text{sen}\, t \end{cases}$$

$$-\frac{\pi}{2} \leq t \leq \pi/2$$

$x=0$



$x=0$
$y+z=4$ ||

$$y^2+z^2=16$$
$$(4-z)^2+z^2=16$$
$$2z^2-8z+\cancel{16}=\cancel{16}$$
$$z^2-4z=0$$
$$z=0 \text{ ou } z=4$$

$$(y^2+z^2)x = \left[(2-2\,\text{sen}\, t)^2+(2+2\,\text{sen}\, t)^2\right]2\sqrt{2}\cos t$$

$$= \left(4-\cancel{8\,\text{sen}\, t}+4\,\text{sen}^2 t+4+\cancel{8\,\text{sen}\, t}+4\,\text{sen}^2 t\right)2\sqrt{2}\cos t$$

$$= (8+8\,\text{sen}^2 t)\,2\sqrt{2}\cos t$$

$$= 16\sqrt{2}\,(1+\text{sen}^2 t)\cos t$$

$$\gamma' : \begin{cases} x' = -2\sqrt{2}\,\text{sen}\,t \\ y' = -2\cos t \\ z' = 2\cos t \end{cases}$$

$$\|\gamma'\|^2 = 8\,\text{sen}^2 t + 4\cos^2 t + 4\cos^2 t = \underline{8}$$

$$\underline{\|\gamma'\| = 2\sqrt{2}}$$

$$\int_C (y^2+z^2)\,x\,ds = \int_{-\pi/2}^{\pi/2} 16\sqrt{2}\,(1+\text{sen}^2 t)\cos t \cdot 2\sqrt{2}\,dt$$

$$= 64 \int_{-\pi/2}^{\pi/2} (\cos t + \text{sen}^2 t \cos t)\,dt$$

$$= 64 \left(\text{sen}\,t + \frac{1}{3}\text{sen}^3 t\right)\Big|_{-\pi/2}^{\pi/2}$$

$$= 64\left[\left(1+\tfrac{1}{3}\right) - \left(-1-\tfrac{1}{3}\right)\right] = 64 \cdot 8/3$$

$$\underline{I_x = 64 \cdot \frac{8}{3}} \qquad M = \int_C x\,ds = \int_{-\pi/2}^{\pi/2} 2\sqrt{2}\cos t \cdot 2\sqrt{2}\,dt$$

$$= 8\,\text{sen}\,t\Big|_{-\pi/2}^{\pi/2} = \underline{16}$$

$$\therefore \quad I_x = 16 \cdot 4 \cdot \frac{8}{3} = M \cdot \frac{32}{3}$$

$$\underline{I_x = \frac{32}{3} M}$$

(8)

$A = (\sqrt{2}, \sqrt{2}, 0)$

$B = (1, 1, \sqrt{2})$

$$C: \begin{cases} x^2 + y^2 + z^2 = 4 \\ y = x \\ x \geq 0, y \geq 0, z \geq 0 \end{cases}$$

C une os ptos A e B

M = massa de C = ?

densidade em cada pto proporcional ao quadrado da distância do pto ao plano YZ, $f(x,y,z) = kx^2$

Solução

$$M = \int_C f\,ds = \int_C kx^2\,ds$$

$y = x$

$$\left.\begin{aligned} x^2 + x^2 + z^2 &= 4 \\ 2x^2 + z^2 &= 4 \\ \frac{x^2}{2} + \frac{z^2}{4} &= 1 \end{aligned}\right\} \quad \left(\frac{x}{\sqrt{2}}\right)^2 + \left(\frac{z}{2}\right)^2 = 1$$

$$\gamma: \begin{cases} x = \sqrt{2}\cos t \\ y = x = \sqrt{2}\cos t \\ z = 2\,\mathrm{sen}\,t \end{cases} \qquad \left.\begin{aligned} x = y = \sqrt{2} \\ z = 0 \end{aligned}\right\} \Rightarrow t = 0$$

$$0 \leq t \leq \pi/4 \qquad \left.\begin{aligned} x = y = 1 \\ z = \sqrt{2} \end{aligned}\right\} \Rightarrow t = \pi/4$$

$$\gamma': \begin{cases} x' = -\sqrt{2}\,\mathrm{sen}\,t \\ y' = -\sqrt{2}\,\mathrm{sen}\,t \\ z' = 2\cos t \end{cases} \qquad \|\gamma'\|^2 = 4, \quad \|\gamma'\| = 2$$

$$M = \int_0^{\pi/4} k \cdot 2\cos^2 t \cdot 2\,dt =$$

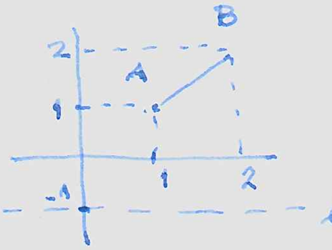
$$M = 4k\int_0^{\pi/4} \cos^2 t \, dt = 4k\int_0^{\pi/4} \frac{1+\cos 2t}{2}\, dt$$

$$= 4k\left(\frac{1}{2}\cdot\frac{\pi}{4} + \frac{1}{2}\cdot\frac{1}{2}\, \text{sen}\, 2t \Big|_0^{\pi/4}\right)$$

$$= 4k\left(\frac{1}{2}\,\frac{\pi}{4} + \frac{1}{4}\right) = k\left(\pi/2 + 1\right)$$

$$\boxed{M = k\,\frac{\pi+2}{2}}$$

9



$A = (1,1)$
$B = (2,2)$

densidade : $k \cdot x$

$I_\ell = ?$

$f = kx \,, \quad d^2 = (1+y)^2$

solução

$$I_\ell = \int_C d^2 f \, ds = \int_C (1+y^2)\, kx \, ds$$

$C: \quad \gamma(t) = (t+1, t+1) \quad \Big\{ \quad \gamma' = (1,1) \,, \quad \|\gamma'\| = \sqrt{2}$
$\quad 0 \leq t \leq 1$

$$I_\ell = \int_0^1 (1+t+1)^2 \cdot k(t+1)\cdot\sqrt{2}\, dt = \sqrt{2}\,k\int_0^1 (t+2)^2 (t+1)\, dt$$

$$= \sqrt{2}\,k\int_0^1 (t^3 + 5t^2 + 8t + 4)\, dt$$

$$= \sqrt{2}\,k\left(\frac{1}{4} + \frac{5}{3} + \frac{8}{2} + 4\right) = \frac{119}{12}\sqrt{2}\,k$$

(10)

$C: \alpha(t) = (e^t \cos t, e^t \operatorname{sen} t, e^t), \quad 0 < t < 1$

densidade inversamente proporcional ao quadrado da distância à origem

$M = ?$ $\quad\quad M = \int_C f \cdot ds$, f = densidade

Solução

$f = \dfrac{k}{x^2+y^2+z^2}$ $\quad \left\{ \begin{aligned} x^2+y^2+z^2 &= e^{2t}\cos^2 t + e^{2t}\operatorname{sen}^2 t + e^{2t} \\ &= 2e^{2t} \end{aligned} \right.$

$\alpha'(t) = (e^t \cos t - e^t \operatorname{sen} t, e^t \operatorname{sen} t + e^t \cos t, e^t)$

$$\|\alpha'\|^2 = e^{2t}\cos^2 t + e^{2t}\operatorname{sen}^2 t \cancel{- 2e^{2t}\operatorname{sen} t \cos t} + e^{2t}\operatorname{sen}^2 t + e^{2t}\cos^2 t \cancel{+ 2e^{2t}\operatorname{sen} t \cos t} + e^{2t}$$

$$= 3e^{2t} \quad\quad \|\alpha'\| = \sqrt{3}\, e^t$$

$$M = \int_0^1 \frac{k}{2e^{2t}} \cdot \sqrt{3}\, e^t \, dt = \frac{\sqrt{3}}{2} k \int_0^1 \frac{1}{e^t} dt$$

$$= \frac{\sqrt{3}}{2} k \left(\frac{-1}{e^t}\Big|_0^1 \right) = \frac{\sqrt{3}}{2} k \left(-\frac{1}{e} + 1 \right)$$

$$= \frac{\sqrt{3}}{2} k \left(1 - \frac{1}{e} \right)$$

11

$$C : \begin{cases} x^2 + y^2 + z^2 = a^2, \ a > 0 \\ x \geq 0 \\ y = z \end{cases}$$

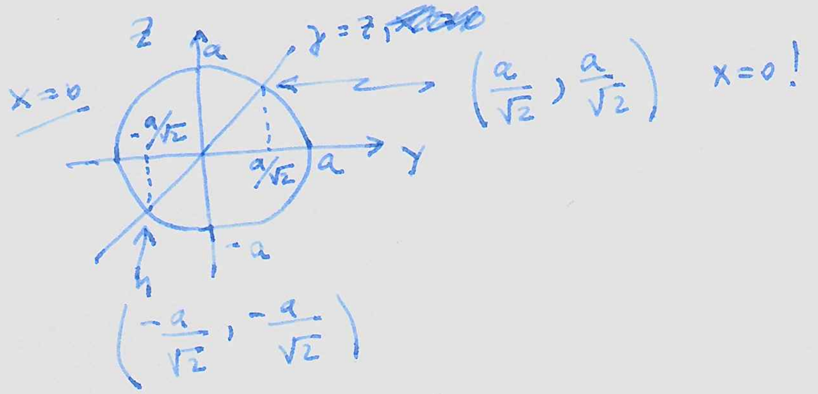
$$\int_C 2xyz \, ds = 16 \qquad \underline{\underline{a = ?}}$$

<u>Solução:</u>

$y = z$

$x^2 + y^2 + y^2 = a^2$

$\underline{x^2 + 2y^2 = a^2}$



$$C) \ \gamma : \begin{cases} x = a\cos t \\ y = \frac{a}{\sqrt{2}} \operatorname{sen} t \\ z = \frac{a}{\sqrt{2}} \operatorname{sen} t \end{cases}$$

$$\left.\begin{matrix} x = 0 \\ y = \frac{a}{\sqrt{2}} \end{matrix}\right\} \Rightarrow \left.\begin{matrix} \cos t = 0 \\ \operatorname{sen} t = 1 \end{matrix}\right\} \Rightarrow t = \pi/2$$

$$\left.\begin{matrix} x = 0 \\ y = -\frac{a}{\sqrt{2}} \end{matrix}\right\} \left.\begin{matrix} \cos t = 0 \\ \operatorname{sen} t = -1 \end{matrix}\right\} \Rightarrow t = -\pi/2$$

$$-\pi/2 \leq t \leq \pi/2$$

$$xyz = \frac{a^3}{2} \operatorname{sen}^2 t \cos t \quad \therefore \quad 2xyz = a^3 \operatorname{sen}^2 t \cos t$$

$$\gamma' : \begin{cases} x' = -a \operatorname{sen} t \\ y' = \frac{a}{\sqrt{2}} \cos t \\ z' = \frac{a}{\sqrt{2}} \cos t \end{cases} \qquad \underline{\|\gamma'\| = a}$$

$$\int_C 2xyz \, ds = \int_{-\pi/2}^{\pi/2} a^3 \operatorname{sen}^2 t \cos t \cdot a \, dt$$

$$= a^4 \left. \frac{\operatorname{sen}^3 t}{3} \right|_{-\pi/2}^{\pi/2} = \underline{\underline{\frac{2}{3} a^4}}$$

$$\therefore \ \frac{2}{3} a^4 = 16 \Rightarrow a^4 = 24$$

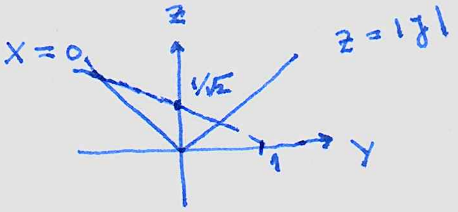
$$\boxed{a = \sqrt[4]{24}}$$

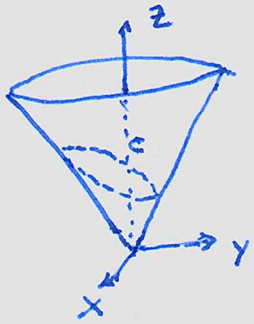
12

$$C : \begin{cases} z = \sqrt{x^2+y^2} \\ y + \sqrt{2}\, z = 1 \end{cases} \qquad f(x,y,z) = |x(y+1)|$$

$$M = \int_C f\, ds = ?$$

Solução





$y = 1 - \sqrt{2}\, z$

$z^2 = x^2 + y^2, \quad z \geq 0$

$z^2 = x^2 + (1-\sqrt{2}\, z)^2 = x^2 + 2z^2 - 2\sqrt{2}\, z + 1$

$\Rightarrow \quad x^2 + z^2 - 2\sqrt{2}\, z + 1 = 0$

$x^2 + (z-\sqrt{2})^2 - 2 + 1 = 0$

$$\underbrace{x^2 + (z-\sqrt{2})^2 = 1}$$

$$\gamma : \begin{cases} x = \cos t \\ y = 1 - \sqrt{2}\, z \\ z = \sqrt{2} + \operatorname{sen} t \end{cases}$$

$$\gamma : \begin{cases} x = \cos t \\ y = -(1+\sqrt{2}\operatorname{sen} t) \\ z = \sqrt{2} + \operatorname{sen} t \end{cases}$$

Note que $z \geq 0 \longrightarrow$

$0 \leq t \leq 2\pi$

$$\gamma' : \begin{cases} x' = -\operatorname{sen} t \\ y' = -\sqrt{2}\cos t \\ z' = \cos t \end{cases}$$

$$\|\gamma'\|^2 = 1 + 2\cos^2 t \quad , \quad \|\gamma'\| = \sqrt{1+2\cos^2 t}$$

$$|x(y+1)| = |\cos t\,(-\sqrt{2}\operatorname{sen} t)| = |-\sqrt{2}\cos t \operatorname{sen} t|$$

$$= \begin{cases} \sqrt{2}\operatorname{sen} t \cos t & \text{se } 0 \leq t \leq \pi/2 \\ -\sqrt{2}\operatorname{sen} t \cos t & \text{se } \pi/2 \leq t \leq \pi \\ \sqrt{2}\operatorname{sen} t \cos t & \text{se } \pi \leq t \leq \frac{3\pi}{2} \\ -\sqrt{2}\operatorname{sen} t \cos t & \text{se } \frac{3\pi}{2} \leq t \leq 2\pi \end{cases}$$

$$\int_C f\,ds = \int_0^{\pi/2} \sqrt{2}\,\text{sen}\,t\cos t\sqrt{1+2\cos^2 t}\,dt$$

$$- \int_{\pi/2}^{\pi} \sqrt{2}\,\text{sen}\,t\cos t\sqrt{1+2\cos^2 t}\,dt$$

$$+ \int_{\pi}^{3\pi/2} \cdots\cdots \quad - \int_{3\pi/2}^{2\pi} \cdots\cdots$$

$$\int \sqrt{2}\,\text{sen}\,t\cos t\sqrt{1+2\cos^2 t}\,dt = -\frac{\sqrt{2}}{4}\left(1+2\cos^2 t\right)^{3/2}$$

$$u = 1+2\cos^2 t$$

$$du = -4\,\text{sen}\,t\cos t\,dt$$

$$\sqrt{2}\int\sqrt{u}\left(-\frac{du}{4}\right) = -\frac{\sqrt{2}}{4}\int\sqrt{u}\,du = -\frac{\sqrt{2}}{6}\,u^{3/2}$$

$$\int_C f\,ds = -\frac{\sqrt{2}}{6}\left[(1+2\cdot 0)^{3/2} - (1+2)^{3/2}\right] + \frac{\sqrt{2}}{6}\left[(1+2)^{3/2} - 1\right]$$

$$- \frac{\sqrt{2}}{6}\left(1-(1+2)^{3/2}\right) + \frac{\sqrt{2}}{6}\left(3^{3/2} - 1\right)$$

$$= \frac{\sqrt{2}}{6}\left(-1+3^{3/2}+3^{3/2}-1-1+3^{3/2}+3^{3/2}-1\right)$$

$$= \frac{\sqrt{2}}{6}\cdot 4\left(3^{3/2}-1\right) = \frac{2\sqrt{2}}{3}\left(3^{3/2}-1\right)$$