Reduced models for water waves in random coastal regions

Large water wave disturbances propagating in the coastal region is a problem of great practical interest and also of many mathematical challenges. From the application side these problems include tsunami interaction with the submerged topography. From the mathematical point of view it is commonly accepted to start with the Euler equations, together with free surface boundary conditions. Then by the asymptotic analysis of the differential operators one is able to get reduced models/PDE systems. In essence one reduces a nonlinear PDE system in two space dimensions to a 1D problem. In some problems it is convenient to model the highly variable disordered topography as a randomly varying component of the domain. The asymptotic analysis of the free surface Euler equations leads to nonlinear dispersive PDE systems, such as the Boussinesq system, but now in the presence of randomly varying coefficients.

In this presentation I will give an overview of the "infinitely many" ways one can deduce different reduced models, for the same wave regime. All these being legitimate, at the physical and asymptotic levels. Nevertheless there is still a great lack of analysis for these variable-coefficient nonlinear Boussinesq-type systems. If time permits I will briefly describe a recent result with an amplitude modulated wave-train, in the presence of a random topography, which leads to a defocusing nonlinear Schrodinger equation.