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EIGENVALUE BOUNDS FOR ASYMMETRIC SHEAR FLOWS

PABLO BRAZ E SILVA* & FÁBIO VITORIANO E SILVA†

Linear stability for general viscous 2D asymmetric shear flows [3]

$$\mathbf{U} = (U(y), 0, 0),\ \mathbf{W} = (0, 0, W(y)),\ y \in (0, 1),$$

is determined by the (dimensionless) equations [2]

$$\begin{aligned} i\alpha\left[(U-c)(D^2-\alpha^2) - U''\right]\widetilde{\psi} &= \left(\frac{1}{R_\mu} + \frac{1}{2R_k}\right)(D^2-\alpha^2)^2\widetilde{\psi} - \frac{R_0}{R_k}(D^2-\alpha^2)\widetilde{w}, \\ i\alpha\left[(U-c)\widetilde{w} - W'\widetilde{\psi}\right] &= \frac{1}{R_\gamma}(D^2-\alpha^2)\widetilde{w} - \frac{2R_0}{R_\nu}\widetilde{w} + \frac{1}{R_\nu}(D^2-\alpha^2)\widetilde{\psi}, \end{aligned} \tag{0.1}$$

where R_γ, R_μ, R_ν, R_k, and R_0 are dimensionless parameters and $D := \frac{d}{dy}$. Let $c = c_r + ic_i$ be any eigenvalue of system (0.1). We discuss the following bounds for c_i, which are analogous to the classical result due to Joseph[1] for flows governed by the Navier-Stokes equations: If $\max\{\frac{R_\mu}{2}, R_k\} < \min\{R_\nu, \frac{R_k}{R_0}\}$, and $\max\{\frac{R_\nu}{2R_0}, R\gamma\} < \min\{\frac{R_\nu}{2}, \frac{R_k}{2R_0}\}$, then

$$c_i \le \frac{q_1+q_2}{2\alpha} - \frac{\pi^2+\alpha^2}{\alpha R},$$

where $\frac{1}{R} := \min\{\frac{1}{R_1} - \frac{1}{R_2}, \frac{1}{R_3} - \frac{2}{R_2}\}$, $q_1 := \max_{y\in[0,1]}|U'(y)|$, $q_2 := \max_{y\in[0,1]}|W'(y)|$. Moreover, there are no amplified disturbances if

$$\begin{cases} \alpha R q_1 < \dfrac{(4,73)^2\pi}{2} + 2^{\frac{3}{2}}\alpha^3, \\ \quad \text{and} \\ \alpha R q_2 < \sqrt{2(\pi^2+\alpha^2)}(4,73)^2, \end{cases} \quad \text{or} \quad \begin{cases} \alpha R q_1 < (4,73)^2\pi + 2\alpha^2\pi, \\ \quad \text{and} \\ \alpha R q_2 < 2\alpha^2\sqrt{\pi^2+\alpha^2}. \end{cases}$$

References

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*Departamento de Matemática , UFPE, PE, Brasil, pablo@dmat.ufpe.br
†Instituto de Matemática e Estatística, UFG, Goiás, Brasil, e-mail: fabio@mat.ufg.br