Elliptic problems with nonlinear terms depending on the gradient and singular on the boundary

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We will consider the problem

$$(P) \qquad \begin{cases} -\Delta u = u^{q\alpha}|\nabla u|^q + \lambda f(x) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain, $1 < q \leq 2$, $\alpha \in \mathbb{R}$ and $f \geq 0$. We prove that:

(1) If $q\alpha < -1$, then problem (P) has a distributional solution for all $f \in L^1(\Omega)$, and all $\lambda > 0$.

(2) If $-1 \leq q\alpha < 0$, then problem (P) has a solution for all $f \in L^s(\Omega)$, where $s > \frac{N}{q}$ if $N \geq 2$, and without any restriction on λ.

(3) If $q = 2$ and $-1 \leq q\alpha < 0$ then problem (P) has infinitely many solutions under suitable hypotheses on f.

(4) If $0 \leq q\alpha$ and $f \in L^1(\Omega)$ satisfies

$$\lambda_1(f) = \inf_{\phi \in W_0^{1,2}(\Omega)} \frac{\displaystyle\int_\Omega |\nabla \phi|^2 dx}{\displaystyle\int_\Omega f\phi^2 dx} > 0,$$

then problem (P) has a positive solution if $0 < \lambda < \lambda_1(f)$ and no positive solution for large λ.

REFERENCE

B. Abdellaoui, D. Giachetti, I. Peral, M. Walias, *Non-linear elliptic problems with dependence on the gradient and singular on the boundary*, Nonlinear Analysis 74 (2011) 1355-1371