

# GLOBAL SOLVABILITY FOR SMOOTH NONSINGULAR VECTOR FIELDS IN THE PLANE

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We address some global solvability issues for classes of smooth nonsingular vector fields  $L$  in the plane related to cohomological equations  $Lu = f$  in geometry and dynamical systems. The main result is that  $L$  is not surjective in  $C^\infty(\mathbb{R}^2)$  if and only if the geometrical condition – the existence of separatrix strips – holds. For nonsurjective vector fields, we demonstrate that if the RHS  $f$  has at most infra-exponential growth in the separatrix strips we can find a global weak solution  $L_{loc}^1$  near the boundaries of the separatrix strips.

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