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# DELAY NONLINEAR BOUNDARY CONDITIONS AS LIMIT OF REACTIONS CONCENTRATING IN THE BOUNDARY

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In recent years, there has been considerable effort devoted to the problem of stabilization and control of partial differential equations through the application of forces on the boundary. The mathematical theory is very complete when the boundary forces are applied with no delays in time. On the other hand when the boundary forces are applied with delays in time in a nonlinear way, not much is known. In order to understand how one can approach such topics one need to understand the dynamics of the closed loop equations.

The goal of this work is to extend the results [1] and [2] to reaction-diffusion problems with delay. We are interested in reaction terms that concentrate in a neighborhood of the boundary and this neighborhood shrinks to boundary, as a parameter $\epsilon$ goes to zero. More precisely, let $\Omega$ be an open bounded set in $\mathbb{R}^n$ with a smooth boundary $\partial\Omega$. We define the strip $\omega_\epsilon = \{x - \sigma \overrightarrow{n}(x) : x \in \partial\Omega \text{ and } \sigma \in [0, \epsilon)\}$, for sufficiently small $\epsilon$, where $\overrightarrow{n}(x)$ denotes the outward normal vector at $x \in \partial\Omega$. Note that it collapses to the boundary when the parameter $\epsilon$ goes to zero. We are interested in the behaviour, for small $\epsilon$, of the solutions of the reaction-diffusion problem with delay in the interior

$$\left\{ \begin{array}{l} \dfrac{\partial u^\epsilon}{\partial t} = \Delta u^\epsilon - \lambda u^\epsilon + \dfrac{1}{\epsilon}\mathcal{X}_{\omega_\epsilon} f\left(u^\epsilon(t), u^\epsilon(t-\tau)\right), \text{ in } \Omega \times (0,\infty) \\ \dfrac{\partial u^\epsilon}{\partial n} = 0, \text{ in } \partial\Omega \times (0,\infty) \text{ and } u^\epsilon = \varphi^\epsilon, \Omega \times [-\tau, 0]. \end{array} \right. \tag{0.1}$$

We proved that these solutions converge to the solution of the parabolic problem with delay in the boundary

$$\left\{ \begin{array}{l} \dfrac{\partial u^0}{\partial t} = \Delta u^0 - \lambda u^0, \text{ in } \Omega \times (0,\infty) \\ \dfrac{\partial u^0}{\partial n} = f(u^0(t), u^0(t-\tau)), \text{ in } \partial\Omega \times (0,\infty) \text{ and } u^0 = \varphi^0, \Omega \times [-\tau, 0] \end{array} \right. \tag{0.2}$$

where $\mathcal{X}_{\omega_\epsilon}$ is the characteristic function of the set $\omega_\epsilon$. Thus the effective reaction in (0.1) is concentrated in $\omega_\epsilon$. Note that this convergence result can be seen as a tool for transfering informations from the interior to the boundary, as so, we then proceed to the study the behavior of the family of global attractors, upper semicontinuity of this family at $\epsilon = 0$ and continuity of the family of equilibria at $\epsilon = 0$. These results can be found in [3] and [4].

## References

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