EBED - Escola Brasileira de Equações Diferenciais
UFPB - Universidade Federal do Paraíba
Edição Nº 4 Agosto 2011

Some linear estimates for a multidimensional version of the Benjamin-Ono equation.

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We address here some properties of the solutions to the equation

$$\partial_t u - (-\Delta)^{1/2}\partial_{x_1} u = 0 \tag{0.1}$$

which constitutes the linear part of

$$\partial_t u - (-\Delta)^{1/2}\partial_{x_1} u + u\partial_{x_1} u = 0,\ x = (x_1, \ldots, x_n),\ \Delta = \partial_{x_1^2} + \partial_{x_2^2} + \cdots + \partial_{x_n^2}. \tag{0.2}$$

When for $n = 1$, (0.2) coincides with the Benjamin-Ono equation

$$\partial_t u - H\partial_x u + u\partial_x u = 0, \tag{0.3}$$

where H denotes the Hilbert transform

$$\widehat{Hf}(\xi) = -isgn(\xi)\hat{f}(\xi). \tag{0.4}$$

For $n = 2$, (0.2) has been derived in [2] as a model that describes the dynamics of 3-D, slightly nonlinear disturbances in boundary-layer shear flow. Soliton solutions for this equation have been studied in [1] and [3].

We will present some estimates that exploits the nature of the symbol of (0.1). Our purpose is to use them to obtain better results about local well-posedness.

References

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