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GLOBAL SOLUTIONS OF A MODEL OF PHASE TRANSITIONS FOR THERMOVISCOELASTIC MATERIALS

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We analyze a family of highly nonlinear systems of partial differential equations including as a particular case the following system:

$$\theta_t + l\chi_t - \Delta\theta = g, \tag{0.1}$$

$$\chi_t - \Delta\chi + W'(\chi) \ni \theta - \theta_c + \frac{|\eta(u)|^2}{2}, \tag{0.2}$$

$$u_{tt} - div((1-\chi)\eta(u) + \chi\eta(u_t)) + \nu(-\Delta)^2 u_t = f, \tag{0.3}$$

subjected to suitable boundary and initial conditions.

These equations may be seen as a model for solidification or melting of certain viscoelastic materials subject to thermal effects and also taking in consideration the possibility of damped vibrations of the solid parts of the material. In this setting, the state variables are the absolute temperature θ, (θ_c being t a given constant equilibrium temperature), an order parameter χ, which is the phase field that in this model stands for the local proportion of the liquid phase in the material, and thus must assume values in the interval $[0,1]$, and also u, which is the vector of the small displacements. W in (0.2) is a given potential which is the sum of a smooth nonconvex function $\widehat{\gamma}$ and of a convex function $\widehat{\beta}$, with domain contained in $[0,1]$ and differentiable in $(0,1)$. The expression $\eta(u)$ denotes the linearized symmetric strain tensor, which in the (spatially) three-dimensional case is given by $\eta_{ij}(u) := (u_{i,x_j} + u_{j,x_i})/2$, $i,j = 1,2,3$ (with the commas we denote space derivatives), while the symbol div stands both for the scalar and for the vectorial divergence operator. Further, the term $(-\Delta)^2$ denotes the biharmonic operator and f on the right-hand side may be interpreted as an exterior volume force applied to the body. The present system is closed related to a model without dissipation proposed by M. Frémond in [2] and analyzed by Rocca and Rossi [3], where existence of local in time solutions (global in the one dimensional case) assuming values just in the mushy zone is proved.

The adaptation of the model in [2], [3], by including suitable dissipation and assuming constant latent heat leads to system (1), for which we are able to prove global in time existence even for solutions that may touch the potential barriers, with correspond to pure solid or pure liquid states.

We remark that one of the difficulties for the analysis is that equation (0.3) may change from parabolic to hyperbolic, with dissipation, and vice-versa depending on the values of the phase field χ; another technical difficulty is that equation (0.2) has a term that acts as a potential barrier and thus involves a multivalued operator.

References

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