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# ENERGY ESTIMATES AT INFINITY FOR HYPERBOLIC-LIKE DISSIPATION EQUATIONS .

MARCELO R. EBERT*

The goal of this talk is to discuss about the long time behavior of the energy to the strictly hyperbolic Cauchy problem

$$\begin{gathered}\partial_t^m u - \sum_{j=1}^{m} a_j(t)\lambda^j(t)\partial_t^{m-j}\partial_x^j u + \sum_{j+k\le m-1} c_{j,k}(t)\lambda^k(t)\partial_t^j\partial_x^k u = 0,\ (t,x)\in(0,\infty)\times\mathbb{R},\\ \partial_t^j u(0,x) = u_j(x), \quad j=0,1,\dots,m-1.\end{gathered} \tag{0.1}$$

For sake of simplicity, we consider this model problem in one space dimension, but our arguments can be immediately extended to the case $x\in\mathbb{R}^n, n\ge 2$.

It is well known (see, e.g., [1]) that if the coefficients are sufficiently regular and bounded, then the Cauchy problem (0.1) is $C^\infty$ well-posed. More precisely, in this case we have well-posedness in Sobolev spaces and for any given Cauchy data $u_j\in H^{s+m-1-j}(\mathbb{R})$, $s\in\mathbb{R}$, there is a unique solution $u\in\bigcap_0^{m-1} C^j([0,\infty);H^{s+m-1-j})$.

The problem of sharp decay estimate goes back to Strichartz-type decay estimate ([2]). Later A. Matsumura ([3]) proved sharp decay estimates for the damped wave equations. They proved the results by using WKB-representation of the solutions. More recently, the influence of a time-dependent coefficient on such decay estimate for the wave equation and dissipative wave equations was studied in a series of papers [4], [5] and [6].

To obtain an energy estimate to the Cauchy problem (0.1), our approach will be, by using the hyperbolicity, to state a very precise energy behavior in some region of the extend phase space, then we state conditions, namely, Hyperbolic-like Dissipation, in order to have the same energy estimate in all the extend phase space.

This is a joint work with Marcello D'Abbicco.

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*Departamento de Computação e Matemática, Universidade de São Paulo, SP, Brasil, ebert@ffclrp.usp.br