EBED - Escola Brasileira de Equações Diferenciais
UFPB - Universidade Federal do Paraíba
Edição Nº 4 Agosto 2011

Liouville-type result for the m-Laplacean

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Consider the following problem

$$-\Delta_m u \geq f(u) \qquad \text{in } \mathbb{R}^N, \tag{0.1}$$

where $1 < m < N$. Mitidieri and Pohozaev proved in [5], among other results, that problem (0.1) has no positive solution provided

$$f(u) \geq cu^p \qquad \text{for all } u > 0, \tag{0.2}$$

where $0 < p < N(m-1)/(N-m)$. We give a simple proof, by using local integral estimates, of a result which implies that the problem (0.1) has no positive solution in the case that f satisfies (0.2) only near of origin.

This kind of results can be used to prove existence results for associated problems in bounded domain. This is particularly useful if the problem under consideration is nonvariational (see for example [1], [6] and the references there in).

Usually these a priori estimates is obtained by using a blow-up technique, together with Liouville-type results (see [2],[3]). Suppose by contradiction, that there exists a sequence of pairs (u_n, τ_n), with u_n unbounded (in the L^∞ norm). Let x_n be a point at which u_n attain their maxima. In the case that f and g are multiple of the same power, the blow-up methods provides a solution of the problem

$$-\Delta_m u \geq u^p,$$

in $\mathbb{R}^N$ or in the halfspace when the points x_n approach sufficiently fast to the boundary of Ω (in comparison with the L^∞ norm of u_n).

As in [4], we employ local integral inequalities together with Harnack-type inequalities to prove that there is no positive solution of $-\Delta_m u \geq f(u)$ in the halfspace.

References

[1] C. Azizieh & P. Clément, *A Priori estimates and continuation methods for positive solutions of p-Laplace equations*, J. Differential Equations, 179(2002), pp. 412-428.

[2] B. Gidas & J. Spruck, *Global and local behaviour of positive solutions of nonlinear elliptic equations*, Comm. Pure and Appl. Math., 34, (1981), pp 525-598.

[3] B. Gidas & J. Spruck, *A priori bounds for positive solutions of nonlinear elliptic equations*, Comm. Partial Differential Equations, 6 (1981), pp 883-901.

[4] S. Lorca, *Nonexistence of Positive Solution for Quasilinear Elliptic Problems in the Half-Space,* J. Inequalities and Applications, 2007, ID 65126.

[5] E. Mitidieri & S.I. Pohozaev, *Nonexistence of positive solutions for quasilinear elliptic problems in* $\mathbb{R}^N$*,* Proc. Steklov Inst. Math., 227(1999), pp. 1-32.

[6] D. Ruiz, *A priori estimates and existence of positive solutions for strongly nonlinear problems,* J. Differential Equations, 199(2004), pp. 96-114.