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The supercritical generalized KdV equation: global well-posedness in the energy space and below.

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We consider the generalized Korteweg-de Vries (gKdV) equation $\partial_t u + \partial_x^3 u + \mu \partial_x (u^{k+1}) = 0$, where $k \geq 5$ is an integer number and $\mu = \pm 1$. In the focusing case ($\mu = 1$), we show that if the initial data u_0 belongs to $H^1(\mathbb{R})$ and satisfies $E(u_0)^{s_k} M(u_0)^{1-s_k} < E(Q)^{s_k} M(Q)^{1-s_k}$, $E(u_0) \geq 0$, and $\|\partial_x u_0\|_{L^2}^{s_k} \|u_0\|_{L^2}^{1-s_k} < \|\partial_x Q\|_{L^2}^{s_k} \|Q\|_{L^2}^{1-s_k}$, where $M(u)$ and $E(u)$ are the mass and energy, then the corresponding solution is global in $H^1(\mathbb{R})$. Here, $s_k = \frac{(k-4)}{2k}$ and Q is the ground state solution corresponding to the gKdV equation. In the defocusing case ($\mu = -1$), if k is even, we prove that the Cauchy problem is globally well-posed in the Sobolev spaces $H^s(\mathbb{R})$, $s > \frac{4(k-1)}{5k}$. This work was partially supported by FAPEMIG and CNPq/Brazil.

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