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GLOBAL WELL-POSEDNESS FOR A CRITICAL PERTURBATION OF THE NONLINEAR SCHRÖDINGER EQUATION.

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We consider the initial value problem for the Schrödinger-Debye system (SD), which appears in nonlinear optics:

$$\begin{cases} iu_t + \frac{1}{2}\Delta u = uv, & t \geq 0,\ x \in \mathbb{R}^N\ (N = 1, 2, 3) \\ \mu v_t + v = \lambda |u|^2, & \mu > 0,\ \lambda = \pm 1, \\ u(x, 0) = u_0(x), \quad v(x, 0) = v_0(x), \end{cases} \tag{0.1}$$

where $u = u(x, t)$ is a complex-valued function, $v = v(x, t)$ is a real-valued function and Δ is the Laplacian operator in the spacial variable.

We present recent global well-posedness results for the SD system with p = 2 (physical case), in the energy space $H^1 \times L^2$ and critical dimension $n = 2$. In particular, we show that, unlike the corresponding *limiting model* ($\mu \to 0$) Cubic Nonlinear Schrödinger equation (CNLS):

$$iu_t + \frac{1}{2}\Delta u = \lambda u|u|^2 \tag{0.2}$$

the SD system is globally well-posed in the focusing case ($\lambda = -1$), without smallness assumption on the initial data.

References

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