Resolução da 1ª Prova

(1) a) Para que $x \in D(f)$, devemos ter $x-1 \neq 0$ e $-x^2+3x-2 \geq 0$

Assim, $x \neq 1$ e $(x-1)(2-x) \geq 0$ Portanto, devemos ter

$x \neq 1$ e $x \in [1,2]$ o que nos dá $x \in (1,2]$.

Portanto $D(f) = (1,2]$

b) • $\lim\limits_{x \to 1^+} f(x) = \lim\limits_{x \to 1^+} \dfrac{\sqrt{-x^2+3x-2}}{x-1} = \lim\limits_{x \to 1^+} \dfrac{\sqrt{(x-1)(2-x)}}{x-1} =$

$= \lim\limits_{x \to 1^+} \dfrac{\sqrt{x-1}\sqrt{2-x}}{x-1} = \lim\limits_{x \to 1^+} \dfrac{\sqrt{2-x}}{\sqrt{x-1}} = \dfrac{1}{0^+} = +\infty$

• $\lim\limits_{x \to 1^-} f(x)$ não existe pois $D(f) = (1,2]$

• $\lim\limits_{x \to 2^+} f(x)$ não existe pois $D(f) = (1,2]$

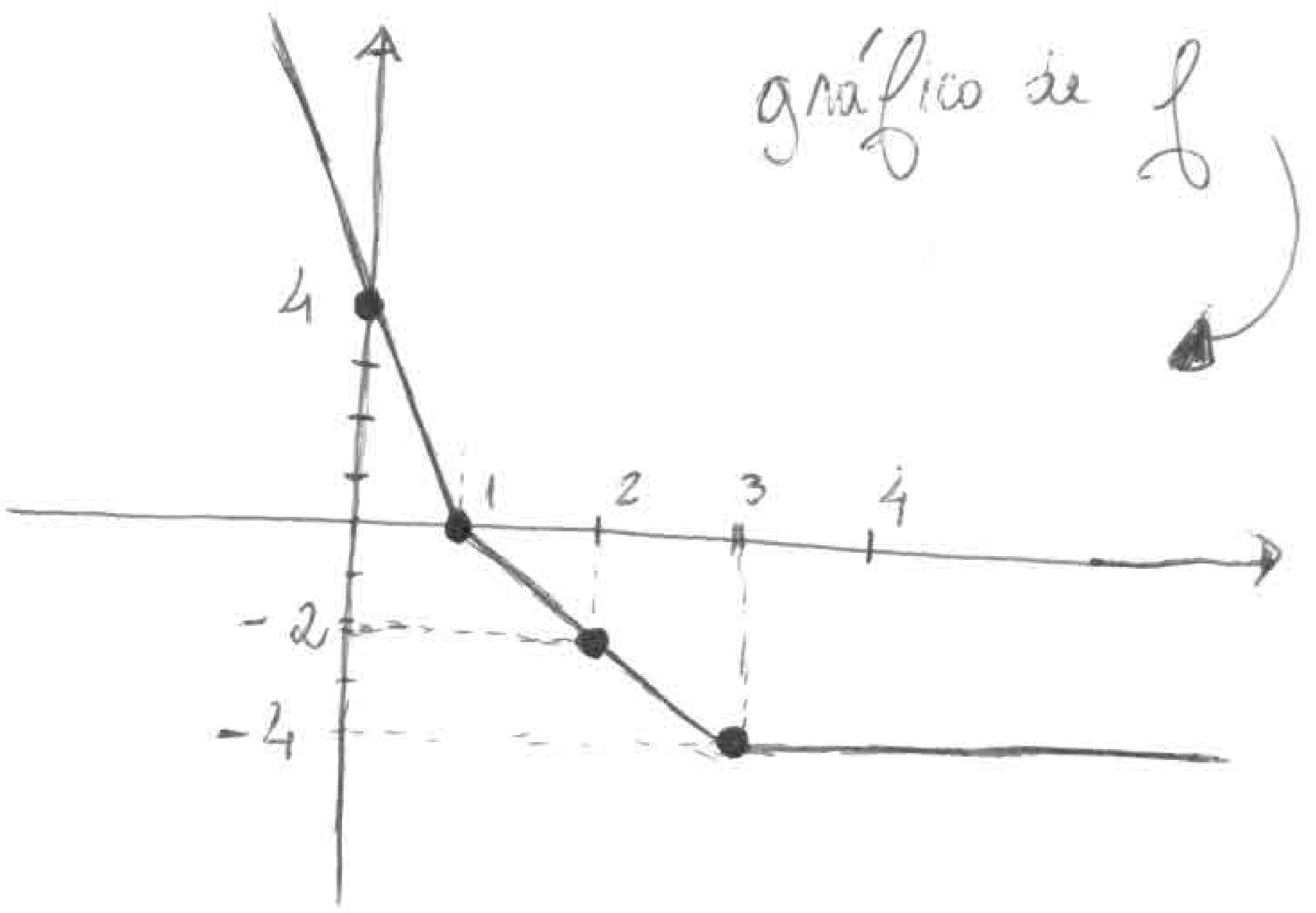
• $\lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^-} \dfrac{\sqrt{-x^2+3x-2}}{x-1} = \dfrac{\sqrt{-2^2+3\cdot 2-2}}{2-1} = \dfrac{0}{1} = 0$

(2) $f: \mathbb{R} \to \mathbb{R}$

$f(x) = |x-1| + |x-3| - 2x$

Logo $f(x) = \begin{cases} 1-x+3-x-2x = -4x+4 & \text{se } x < 1 \\ x-1+3-x-2x = -2x+2 & \text{se } 1 \leq x < 3 \\ x-1+x-3-2x = -4 & \text{se } x \geq 3 \end{cases}$

esboço do gráfico:



Se $x=0$ $\quad f(x) = -4\cdot 0+4 = 4$

Se $x=1$ $\quad f(x) = -2\cdot 1+2 = 0$

Se $x=2$ $\quad f(x) = -2\cdot 2+2 = -2$

se $x \geq 3$ $\quad f(x) = -4$

(2) Seja $g: \mathbb{R} \to \mathbb{R}$ tal que $g(x) = 3x-5$

Queremos $f: \mathbb{R} \to \mathbb{R}$ tal que $g(f(x)) = x$. Ora,

$$x = g(f(x)) \Rightarrow x = 3f(x) - 5 \Rightarrow f(x) = \frac{x+5}{3}$$

Portanto $f: \mathbb{R} \to \mathbb{R}$ é tal que $f(x) = \dfrac{x+5}{3}$.

Com isto $$f(g(x)) = \frac{g(x)+5}{3} = \frac{3x-5+5}{3} = \frac{3x}{3} = x,$$

como queríamos mostrar.

(3) a) $$\lim_{x\to 1} \frac{\sqrt{x^2-7x+10}-2}{x^2-3x+2} = \lim_{x\to 1} \frac{\left(\sqrt{x^2-7x+10}-2\right)\left(\sqrt{x^2-7x+10}+2\right)}{(x-1)(x-2)\left(\sqrt{x^2-7x+10}+2\right)}$$

$$= \lim_{x\to 1} \frac{x^2-7x+10-4}{(x-1)(x-2)\left(\sqrt{x^2-7x+10}+2\right)} = \lim_{x\to 1} \frac{\cancel{(x-1)}(x-6)}{\cancel{(x-1)}(x-2)\left(\sqrt{x^2-7x+10}+2\right)}$$

$$= \frac{-5}{-1\left(\sqrt{1-7+10}+2\right)} = \frac{5}{4}$$

b) $$\lim_{x\to 0} \frac{\tan x}{x} = \lim_{x\to 0} \frac{\operatorname{sen} x / \cos x}{x} = \lim_{x\to 0} \left(\frac{\operatorname{sen} x}{x}\right)\cdot \frac{1}{\cos x}$$

$$= \left(\lim_{x\to 0} \frac{\operatorname{sen} x}{x}\right)\cdot\left(\lim_{x\to 0} \frac{1}{\cos x}\right) = 1 \cdot \frac{1}{\cos 0} = 1$$

(4) Calculemos

$$\lim_{x\to 1} \frac{-x^3+4x^2-3x}{x-1} \quad e \quad \lim_{x\to 1} \frac{x^2-1}{x-1}$$

- $$\lim_{x\to 1} \frac{-x^3+4x^2-3x}{x-1} = \lim_{x\to 1} \frac{\cancel{(x-1)}(-x^2+3x)}{\cancel{x-1}} = -1+3 = 2$$

Divisão de polinômios :

$$\begin{array}{r|l} -x^3+4x^2-3x & x-1 \\ \underline{+x^3-x^2} & -x^2+3x \\ 3x^2-3x & \\ \underline{-3x^2+3x} & \\ 0 & \end{array} \Rightarrow -x^3+4x-3x = (x-1)\cdot(-x^2+3x)$$

- $$\lim_{x\to 1} \frac{x^2-1}{x-1} = \lim_{x\to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} = 1+1 = 2$$

Assim, pelo teorema do confronto, temos $\lim\limits_{x\to 1} g(x) = 2$

Agora note que $$\lim_{x\to +\infty} \frac{-x^3+4x^2-3x}{x-1} = \lim_{x\to+\infty} \frac{x^3(-1-4/x-3/x^2)}{x^3(1/x^2-1/x^3)} = \frac{-1}{0^+} = -\infty$$

$$\lim_{x\to +\infty} \frac{x^2-1}{x-1} = \lim_{x\to+\infty} x+1 = +\infty.$$

Sendo assim, nada podemos concluir a respeito de $\lim\limits_{x\to+\infty} g(x)$.

(5) $f:\mathbb{R}\to\mathbb{R}$ tal que $f(x) = x^2+3x$. Logo

$$\lim_{t\to 0} \frac{f(x_0+t)-f(x_0)}{t} = \lim_{t\to 0} \frac{(x_0+t)^2+3(x_0+t)-x_0^2-3x_0}{t} =$$

$$= \lim_{t\to 0} \frac{x_0^2+2x_0t+t^2+3x_0+3t-x_0^2-3x_0}{t} = \lim_{t\to 0} \frac{2x_0t+t^2+3t}{t}$$

$$= \lim_{t\to 0} \frac{2x_0+t+3}{1} = \lim_{t\to 0} 2x_0+t+3 = 2x_0+3.$$

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